THE REVERSAL OF THE HALL EFFECT IN ALLOYS.

By Alpheus W. Smith.

IN studying the Hall effect in bismuth-tin alloys von Ettingshausen and Nernst[1] made the interesting observation that for a certain strength of the magnetic field there is a reversal in the direction of the Hall electromotive force. For small values of the magnetic field the direction of this electromotive force is the same as that in bismuth. As the magnetic field is increased the Hall electromotive force at first increases, passes through a maximum, and sinks to zero, after which it reverses its direction and increases continuously. To account for this reversal von Ettingshausen and Nernst put forth the following suggestion which they did not undertake to verify.

In Fig. 1 let the plate AB have a current of electricity flowing from A to B. If there is a magnetic field perpendicular to the plane of the plate, two phenomena are observed simultaneously. If the direction of the flow of the Amperian magnetizing current is that indicated by the arrow on the circle, the upper edge of the plate becomes negative with respect to the lower and its temperature is increased above that of the lower. There is thus set up a temperature gradient from the top to the bottom of the plate. If the plate is perfectly insulated the flow of heat produced by this temperature gradient is just balanced by the heat generated at the top in excess of that at the bottom of the plate. The difference of temperature thus established is a measure of the von Ettingshausen effect and is given by the equation

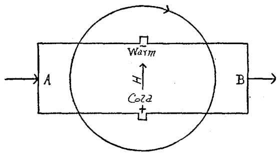

Fig. 1.

$$\Delta t = {}_eT_h \frac{Hi}{d},$$

where i is the current in absolute units, H the magnetic field and d the thickness of the plate in centimeters. The Hall electromotive force is given by the equation,

$$E = {}_eT_e \frac{Hi}{d}.$$

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[1] Ann. der Phys., 33, p. 474, 1888.

If it happens that the plate is imperfectly insulated from its surroundings there will be a flow of heat from the medium into the plate. This flow will enter the bottom where the plate is colder than the surroundings and leave it at the top where it is warmer than the surroundings. It seems to me that a measure of the maximum flow to be expected from this source may be had by regarding the actual temperature gradient in the plate reversed in direction, so that there is in the plate from bottom to top a flow of heat in the direction opposite to that which would result from the observed temperature gradient in the plate. This flow is indicated by the vertical arrow in Fig. 1.

It has been observed in these same alloys that if the flow of electricity is replaced by a flow of heat as indicated in Fig. 2 and if the thermo-electromotive force between two points A and B on the longitudinal axis of the plate is compensated, under the magnetic action there is set up a difference of potential between these points. Of course this potential difference may be interpreted as a change in thermo-electromotive force on account of the magnetic action. This difference of potential is not reversed with the magnetic field. In bismuth and these alloys it is found that its direction is such that the hot end of the plate is positive with respect to the cold end. The magnitude of this potential difference is given by the equation,

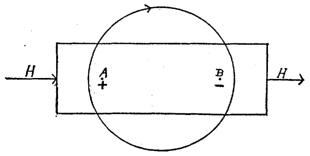

Fig. 2.

$$E = LH\frac{\partial t}{\partial x},$$

where $\partial t/\partial x$ is the temperature gradient in the plate.

Now suppose that on account of the transverse flow of heat which von Ettingshausen and Nernst assume to be associated with the Hall effect, there is established this longitudinal effect between the top and the bottom of the plate. This would give an electromotive force which would be superposed on the ordinary Hall electromotive force. These two electromotive forces would be opposite in direction.

The observed electromotive force would be their algebraic sum. Let E = the Hall electromotive force between the top and bottom of a plate one centimeter in thickness when a current of one electromagnetic unit is flowing in it. Let Δt = the corresponding difference of temperature between the top and bottom of a plate one centimeter wide. Let E_2 = the longitudinal potential difference established in the plate by the magnetic action when the temperature gradient is one degree per cm.

Let E_1 = observed Hall electromotive force. Then,

$$E_1 = E - E_2\Delta t.$$

Whether this will account for the reversal is obviously a question of the relative magnitudes of these potential differences.

More recently Becquerel[1] observed that it is possible to obtain this reversal of the Hall electromotive force in a plate cut from a crystal of bismuth. In such plates the magnetic field at which the reversal takes place depends on the direction of the crystalline axis with respect to the magnetic field and the plane of the plate. Chapman[2] has observed that the Corbino effect is reversed in these bismuth-tin alloys under essentially the same conditions under which the Hall electromotive force is reversed. In the Corbino effect the plates are discs. The current enters at the center of the disc, flows along the radius and leaves at the periphery. The magnetic action causes the current to have a component normal to the radius of the disc. The effective electromotive force producing this component of the current is essentially a Hall electromotive force. Under the conditions under which it is produced it is not clear that the temperature effect to which von Ettingshausen and Nernst attribute the reversal can arise.

The author has observed that the Hall effect is reversed in bismuth-tin alloys for alternating currents with a frequency of 60 cycles per sec. It would seem that for alternating currents of this frequency the accompanying thermal effects on which the explanation of von Ettingshausen and Nernst is based, would scarcely have sufficient time to become effective. The fact that the reversal occurs under these conditions shows that either the explanation is not to be ascribed to the accompanying thermal effects or that these thermal effects manifest themselves more quickly than is ordinarily supposed.

In view of these considerations it seemed worth while to study the reversal of the Hall effect in these alloys, extending the observations to higher field than those used by von Ettingshausen and Nernst. The reversal has also been observed in alloys of bismuth and lead. The principal point of interest is to see whether this reversal of the Hall effect as ordinarily defined can be caused by the superposition of some of the allied phenomena on the Hall electromotive force.

The continuous curves in Fig. 3 which have been plotted from the observations of von Ettinghausen and Nernst show the relation between the observed Hall electromotive force and the magnetic field for two

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[1] Comp. Rendus, 154, pp. 1795–1798, 1912.

[2] Phil. Mag., 32, pp. 303–326, 1916.

alloys of bismuth and tin. The ordinates are the Hall electromotive forces in a plate one cm. thick with a current of one electromagnetic unit in it. One of these alloys contained 0.95 per cent. tin; the other contained 1.46 per cent. tin. The dotted curves in these figures show the potential difference to be expected from the thermal flow assumed by von Ettingshausen and Nernst. It must be remembered that the measure of this flow of heat is obtained by assuming the temperature difference between the top and bottom of the plate, arising from the magnetic action on the electric current, is reversed. This flow is then at right angles to the direction in which the electric current is flowing. On account of this flow of heat the magnetic field produces a difference of potential between the top and bottom of the plate which would be either added to, or subtracted from the Hall electromotive force. The ordinates for these curves are obtained by multiplying the longitudinal potential difference for a particular magnetic field when the temperature gradient is one degree per cm. in the plate by the difference of temperature between the top and the bottom of the plate due to the Ettingshausen effect when the plate is one centimeter wide and one centimeter thick and is traversed by a current of one electromagnetic unit. It is obvious from these curves that these longitudinal potential differences are much too small to account for the observed reversal of the Hall electromotive force.

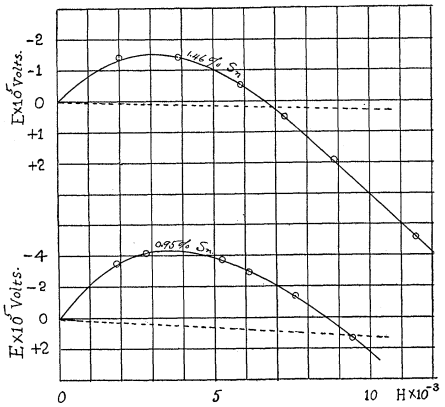

Fig. 3.

Since the properties of tin and lead are very similar it seemed probable that alloys of bismuth and lead would show the reversal of the Hall electromotive force which had been observed by von Ettingshausen and Nernst in alloys of bismuth and tin. In order to test this, three alloys of bismuth and lead were prepared by fusing together in known proportions by weight Baker's analyzed bismuth and Kahlbaum's pure lead. These alloys were then cast in the form of rectangular plates about 0.1 cm. thick, 1.5 cm. wide, and 4.0 cm. long, with narrow arms projecting from the middle of each of the longer sides of the rectangle. These arms were sufficiently long to project outside of the more intense part of the magnetic field. To these arms were soldered the lead wires which were joined to the galvanometer on which the Hall electromotive force was observed. By filing these arms near the edge of the plate they could be shifted until they were nearly on the same equipotential line. Heavy strips of copper were soldered along the ends of the plate and served as electrodes by which the current entered or left the plate. Care was taken that the lines of flow be as nearly as possible parallel to the edges of the plate. With such a plate it is unnecessary to make correction for the thermo-electromotive force arising from the temperature difference between the top and bottom of the plate set up by the magnetic action, for the lead wires where they are joined to the plate are of the same material as the plate and the temperature difference which results from the magnetic action will not produce an appreciable thermo-electromotive force. Essentially no temperature change occurs where the arms are soldered to the lead wires. This form of plate avoids the necessity of making that correction for the Ettingshausen effect which is necessary where the lead wires are soldered directly to the plate.

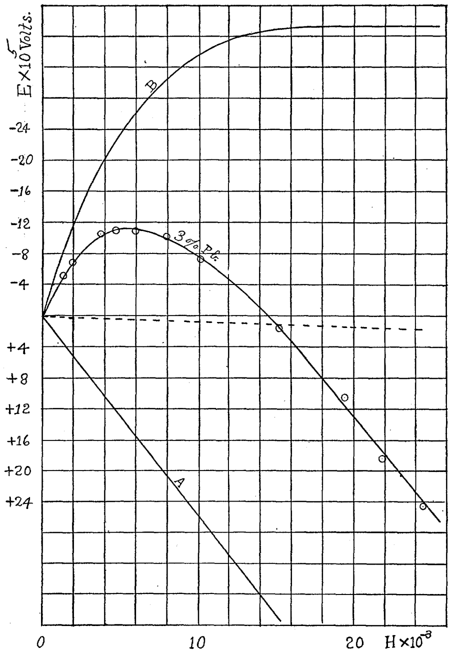

Fig. 4.

For further details with respect to the measurement of the Hall effect in these alloys reference is made to an earlier paper on "The Variation of The Hall Effect in Metals with Change of Temperature."[1] Except for minor details the arrangement of the apparatus and the method of taking observations were the same as used in that paper.

In order to know whether the idea suggested by von Ettingshausen and Nernst would account for the reversal of the Hall effect which was observed in these alloys of bismuth and lead, it was necessary to know the Ettingshausen effect and the longitudinal potential difference in them. Observations had already been made on the Ettingshausen effect in these alloys in connection with the study of this effect in several series of alloys. A description of the method used and of the results obtained

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[1] PHYS. REV., 30, p. 1, 1910.

in that investigation were published in the Physical Review, N. S., Vol. VIII., p. 82, 1916. The necessary data for the present purpose were taken from the results given in that paper.

For the investigation of the longitudinal thermomagnetic potential difference the same method was used which the author has previously described in the Physical Review, N. S., Vol. II., p. 383, 1913. In the present investigation no essential departures from the details of that method were made.

The curves showing the results obtained in the study of these three alloys of bismuth and lead have been given in Figs. 4, 5 and 6. These alloys contain 3 per cent., 5 per cent., and 10 per cent. of lead respectively. From the observations on the Ettingshausen effect and on the longitudinal potential difference, the potential differences to be expected from the transverse flow of heat assumed by Ettingshausen and Nernst, have been calculated. These potential differences have been plotted for ordinates in the dotted curves given in Figs. 4, 5 and 6. Here as in the alloys studied by von Ettingshausen and Nernst the potential differences which might arise from the longitudinal potential effect are much too small to account for the reversal of the Hall effect.

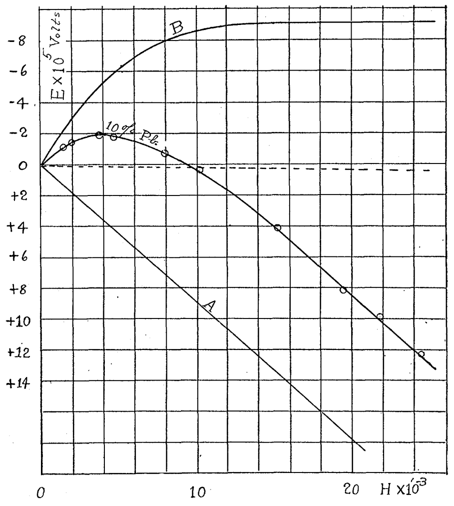

Fig. 5.

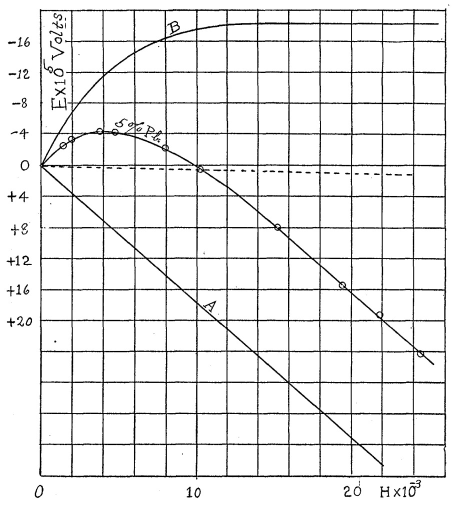

Fig. 6.

Becquerel has suggested that such curves, showing the relation between the Hall electromotive forces and the magnetic fields may be split up into two curves,—Curve A, a straight line passing through the origin and Curve B which arises to a fixed value after which it is parallel to the horizontal axis. The sum of the ordinates of these two curves gives the ordinates of the observed curve. This analysis regards the Hall effect as made up of two parts which are opposite in sign. Of course this analysis is arbitrary, as a number of other pairs of curves may be chosen. It is, however, suggested by the fact that for the larger values of the magnetic field the Hall electromotive force is proportional to the magnetic field. Furthermore, this analysis is helpful in the present discussion because it shows the least correction that must be added to the Hall electromotive force to account for the observed reversal. The slope of Curve *A* is more than forty times that of the corresponding dotted curve showing the possible correction arising from the longitudinal potential difference. Even admitting the validity of the assumption made by von Ettinghausen and Nernst concerning the reversed flow of heat it is necessary to conclude that their suggestion cannot account for the reversal of the Hall effect.

In Fig. 7 the Hall electromotive forces in bismuth and in three alloys of bismuth and lead have been plotted against the magnetic fields producing them. In Fig. 8, similar curves are shown for bismuth and two alloys of bismuth and tin. The characteristics of each of these sets of curves are essentially the same. Small additions of either lead or tin to bismuth causes a rapid decrease in the Hall effect. For any one of the alloys the Hall electromotive force at first has the same direction as in

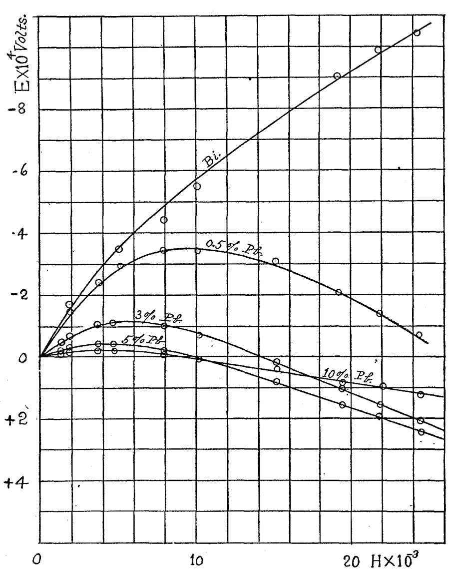

Fig. 7.

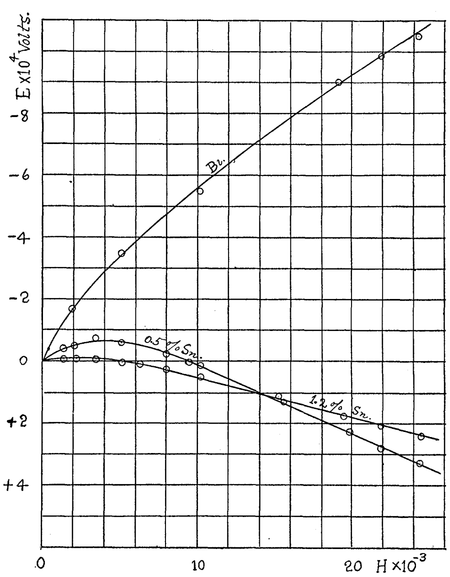

Fig. 8.

bismuth. With increasing field it increases to a maximum, reverses its direction and then increases proportional to the magnetic field.

It is necessary to conclude from these and other observations that there has yet been no satisfactory suggestion advanced to account for the reversal of the Hall effect under the conditions considered in this paper. The explanation of this interesting fact is probably to be looked for in the structure of the alloy, particularly in the interstices between the vibrating atoms.

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