PROCEEDINGS

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AMERICAN PHYSICAL SOCIETY.

RADIATION AND ATOMIC STRUCTURE.[1]

BY R. A. MILLIKAN.

WHILE the study of the physical and chemical properties of matter has produced our present atomic theory and furnished most of the information which is available about the way in which the myriad molecular structures are built up out of their atomic constituents, it has been chiefly the facts of radiation which have provided reliable information about the inner structure of the atom itself. Indeed, during all the years in which the dogma of the indestructible and indivisible atom was upon the stage, it was the complexity of the spectra even of simple gases which kept the physicist in the path of truth, and caused him continually to insist that the atom could not be an ultimate thing, but rather that it must have a structure and a very intricate one at that—as intricate, in Rowland's phrase, as a grand piano.

Yet the evidence of spectroscopy, though tremendously suggestive in the series relationships brought to light between the frequencies of the different lines of a given substance, was after all most disappointing in that it remained wholly uninterpreted in terms of any mechanical model. No vibrating system was known which could produce frequencies related in the manner corresponding to the frequencies found even in the simplest of series, viz., the Balmer series of hydrogen. The discovery and study in the late nineties of corpuscular radiations of the alpha and beta type, with the changes in chemical properties accompanying them, merely served to confirm the century-old evidence of the spectroscope as to the fact of the complexity of the atom, and to educate the public into a readiness to accept it, without at first adding much information as to its nature. These studies did reveal, however, two types of bodies, the alpha and beta particles, as atomic constituents, though they said nothing at first as to their number, their arrangement, or their condition within the atom.

It was the study by Barkla of a radiation problem, namely the problem of the secondary X-radiations scattered by atoms, which furnished the first important evidence as to the number of electronic constituents within an atom. He found

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[1] Presidential address delivered at the New York meeting of the Physical Society, December 27, 1916.

that the number of electrons which can act as scattering centers for X-rays is about half the atomic weight.[1] This conclusion was brilliantly confirmed by the simultaneous study in the Manchester laboratory of the scattering of the alpha rays in passing through matter,[2] and out of the converging evidence of these two types of research there emerged with considerable definiteness the Rutherford nucleus atom, consisting of a central, positively charged body of extraordinarily minute dimensions, its diameter being not over a ten thousandth of the diameter of the atom, surrounded, in the outer regions of the latter, by a number of negative electrons equal to about half the atomic weight. In this statement " the diameter of the nucleus " means the diameter of that portion of the atom which is found by experiment to be impenetrable to the alpha rays, while the diameter of the atom means the average distance of approach of the centers of two atoms in thermal encounters.

But it was again the study of a radiation problem which had to be called upon to furnish unquestionable information as to the exact value of this number, and at the same time to provide the most convincing evidence that we have of the general correctness of the conception of the nucleus atom.

In a research[3] which is destined to rank as one of the dozen most brilliant in conception, skillful in execution, and illuminating in results in the history of science, a young man but twenty-six years old threw open the windows through which we can now glimpse the sub-atomic world with a definiteness and certainty never even dreamed of before. Had the European war had no other result than the snuffing out of this young life, that alone would make it one of the most hideous and most irreparable crimes in history.

For the proof that there exist but 92 elements, from the lightest known one, hydrogen, to the heaviest known one, uranium, and that these are built up one from the other by the successive addition of one and the same electrical element to the nucleus, this proof comes alone from Moseley's discovery (checked and extended as it has been by de Broglie clear up to uranium) that the square roots of the characteristic X-ray frequencies of the elements progress by almost exactly equal steps from the lightest observable one to the heaviest. Moseley proved this in a general way for both the alpha and the beta emission lines of the hardest characteristic X-rays of the elements, the so-called K-rays, and also for the alpha and beta lines of the next softest series, the L series. Fig. 1 shows the now familiar regular progression of the wave-lengths of both the K and the L lines, as the elements which produce them rise in atomic weight and atomic number from top to bottom of the figure. These photographs, which are due to Siegbahn, are introduced merely to pave the way for the discussion and to enable comparison with Fig. 2, which represents the absorption spectra in various substances of the radiation emitted by tungsten. These beautiful photographs have just been sent me by de Broglie.[4] They are

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[1] Barkla, Phil. Mag., 21, 648, May, 1911.

[2] Rutherford, Phil. Mag., 21, 669, May, 1911.

[3] Moseley, Phil. Mag., 26, 1024, Dec., 1913, and 27, 703, 1914.

[4] See Compt Rendu, 165, 87, 352, 1917.

taken by de Broglie's own method of the continuous rotation of the crystal of the X-ray spectrometer.

They show the general radiation of tungsten as a background in all the photographs, and the two K lines of tungsten (W) also in all, while the L series of tungsten appears in the upper one. The edge of the band marked $\frac{\mathrm{Ag}}{\mathrm{K_A}}$ is the exact point at which, with increasing frequency, the general radiation constituting the background begins to be absorbed by the silver atoms (atomic number 47) in the photographic plate, and the point for all frequencies above which it is absorbed in the remarkable manner discovered by Barkla to be characteristic of the absorption of X-rays. The edge marked $\frac{\mathrm{Br}}{\mathrm{K_A}}$ is the corresponding point for the bromine of the photographic plate whose atomic number is 35. These two points, characteristic of the AgBr emulsion of the plate, appear on most of the photographs. Absorption of course appears in the photographic plate as a lightening of the background, elsewhere as a darkening. The way in which this outer edge of the absorption band moves toward the central image as the atomic number increases in the steps Br 35, Mo 42, Ag 47, Cd 48, Sb 51, Ba 56, W 74, Hg 80, is very beautifully shown, in de Broglie's photographs, clear up to mercury, where the absorption edge is somewhat inside the shortest of the characteristic K radiations of tungsten. This latter line coincides, nearly if not exactly as will be shown in Table I., with the absorption edge of tungsten. There must be 12 more of these edges between mercury and uranium, and de Broglie has measured them clear up to thorium, thus extending the K series from N = 60 to N = 90, an enormous advance. The absorption edges become, however, very difficult to locate in the K region of frequencies because of their extreme closeness to the central image. But Fig. 3 shows the L-ray absorption bands in uranium and thorium. Fig. 4 shows how closely these absorption edges follow the Moseley law of equal steps for as many as twenty steps. In going from bromine, atomic number 35, to uranium, atomic number 92, the length of the Moseley step does change however by a few per cent. The data given in Table I. bring out the exceedingly interesting relation that the absorption edge coincides exceedingly closely in every case with the shortest emission K line of the absorbing substance, while in the L series one of the two absorption edges coincides exactly in every case with the shortest emission beta ray of the L series. The other coincides also in every case with an emission line, though the data is yet too meagre to permit of a generalization in the case of this second L absorption band.

Now it is these radiating and absorbing properties of atoms and these alone which justify a series of atomic numbers differing from and more fundamental than the series of atomic weights. Our present series of atomic numbers is simply this Moseley series of steps based on square root frequencies. It is true that a series of atomic numbers coinciding with the series of atomic weights was suggested earlier, indeed 100 years earlier, by Prout, and by many others since then, and it is true, too, that changes in the chemical properties of radioactive substances accompanying the loss of alpha and beta particles led van den Broek,[1] just before Moseley's work appeared, to suggest that position in the periodic table might be a more fundamental property than atomic weight,[2] but since this position is in some instances uncertain, and since the number of elements was wholly unknown, no definite numbers were or could be assigned to all the elements until Moseley's discovery was made, and the only evidence which we now have as to just how many elements there are between hydrogen and uranium, and as to just where each one belongs, is the evidence of the X-ray specta. It is true that between helium, atomic number 2, and sodium, atomic number 11, we have no evidence other than the order of atomic weights, the progression of chemical properties, and the number of known elements in this region, to guide us in completing the table, but since in the region of low atomic weights the progression in the Moseley table is always in agreement with the progression in the periodic table there can be little doubt of the correct number of each element even in this region which is as yet inaccessible to X-ray measurements. Moseley's name must then be set over against one of the most epoch making of the world's great discoveries. And I wish to call attention to some important conclusions as to atomic structure which are rendered extremely probable by it.

Table I.

Comparison of K_A and K_β.

N.	**Element.**	K_A.	K_β.	*N.*	**Element.**	K_A.	K_β.
35	Br	.914	.914	53	I	.367	(.380)
37	Rb	.810	.813	55	Cs	.338	(.345)
38	Sr	.764	.767	56	Ba	.325	(.333)
40	Zr	.681	(.695)	57	La	.310	(.319)
41	Nb	.645	.657	58	Ce	.298	.(304)
42	Ma	.611	(.620)	78	Pt	.150	
46	Pd	.503	.503	79	Au	.147	
47	Ag	.479	.488	80	Hg	.143	
48	Cd	.458	(.466)	81	Tl	.139	
50	Sn	.419	(.419)	82	Pb	.135	
51	Sb	.399	.408	83	Bi	.130	
52	Te	.383	(.396)	90	Th	.098 about	

Comparison of L_A and L_β.

N.	**Element.**	L_A.	L_β.	*N.*	**Element.**	L_A.	L_β.
78	Pt	1.067	1.072	90	Th	.756	.750
79	Au	1.037	1.035	92	U	.719	.702
82	Pb	.945	.948				

The first is this. If we may assume that the ordinary law of inverse squares holds for the forces exerted by the atomic nucleus on negative electrons near

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[1] Van den Broek, Phys. Zeil., XIV., 32, 1913.

[2] This however was not a new suggestion, see, for example, A. J. Hopkins, J. Am. Ch. S., 1027, 1911.

it—and this time-honored law, so amply verified in celestial regions, has been fully verified for sub-atomic regions as well by the work done at the University of Manchester on the scattering of alpha rays—then the Moseley law that the square roots of the highest frequencies obtainable from different atoms are proportional to the nuclear charge[1] means, without any quantum theory, that the distances from the nucleus of each type of atom to the orbit of the inmost electron, if there be such an orbit, is inversely proportional to the charge on the nucleus, *i. e.*, to the atomic number. To see this it is only necessary to apply the Newtonian law connecting central force eE, orbital frequency n and radius a, namely

$$\frac{eE_1}{a^2} = (2\pi n)^2 ma, \quad \text{or} \quad \frac{n_1^2}{n_2^2} = \frac{E_1 a_2^3}{E_2 a_1^3} \tag{1}$$

and then to set as the statement of Moseley's experiment

$$\sqrt{\frac{n_1}{n_2}} = \frac{E_1}{E_2} \tag{2}$$

when there results at once from (1) and (2)

$$\frac{E_1}{E_2} = \frac{a_2}{a_1} \tag{3}$$

It may be objected that in the setting up of these relations I have made two assumptions, the one that the electrons rotate in circular orbits, and the other that the observed highest frequencies are proportional to the highest orbital frequencies. The first assumption is justified (*a*) by the fact that the recognized and tested principles of physics give us no other known way of providing a stable system, (*b*) by the experimental facts of light (Zeeman effect) and (*c*) by the phenomena of magnetism, especially the recent ones brought to light by Einstein and de Haas,[2] and by Barnett,[3] which well-nigh demonstrate the existence of permanent and therefore non-radiating electronic orbits. The exact circular form for the orbit is a secondary matter upon which, as will appear later, it is not necessary to insist. The second assumption, that the frequencies of the corresponding emission lines in the spectra of the various atoms are proportional to the orbital frequencies, is from a priori considerations probable and from certain theoretical considerations to be presented later necessary.

A second conclusion may be drawn from Moseley's discovery that the L lines progress in frequency from element to element just as do the K lines, the frequency being in each case between one seventh and one eighth as great. It is that, if there is a first or inmost electronic orbit, there must also be a second

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[1] This is the proper statement of the Moseley law, as he himself *interpreted* his results. He knew and was careful to state that there is not an exact linear relation between the atomic number and the square roots of the frequencies, but the lack of exactness of (2) both as to straightness and to intercept may well be attributed to secondary causes (see below).

[2] Verh. d. D. Phys. Ges., XVII., p. 152, 1913.

[3] PHYS. REV., 6, 239, '15; also PHYS. REV., July, 1917.

one in all elements the radius of which is given by (1) to be about $(8)^{2/3}$ or four times as great as that of the first, provided orbital frequencies are in this case too assumed proportional to observed frequencies.[1]

Guided then by the newly discovered facts of X-radiations, and the unquestioned laws of force between electric charges, we get our first information as to the probable positions and conditions of some at least of the negative electrons within the atom.

Again, having found the highest natural frequency which can come from any element, viz., that from uranium, it is of extraordinary interest to inquire where, according to Moseley's law (2), the highest frequency line of the K series would fall for the lightest known element, hydrogen, whose nucleus should consist of but a single positive electron. This is obtained as shown in (2) by dividing the observed highest frequency of any element by the square of the atomic number. The shortest wave-length given out by tungsten, atomic number 74, is, according to de Broglie's measurements, $.177 \times 10^{-8}$ cm., and according to Hull's measurements, $.185 \times 10^{-8}$ cm. This gives for the shortest wave-length which could be produced by hydrogen, according to de Broglie $.177 \times 10^{-8} \times 74^2 = 97.9 \mu\mu$, and according to Hull $101.3 \mu\mu$. This is as close as could be expected, in view of the uncertainties in the measurements and the further fact that Moseley's steps are not quite exact, to the head of the ultra-violet series of hydrogen lines recently discovered by Lyman and located at $91.2\ \mu\mu$. There is every reason to believe, too, from the form of Balmer's series, of which this is the convergence wave-length, that this wave-length corresponds to the highest series frequency of which the hydrogen atom is capable. *It is fairly certain, then, that this Lyman ultra-violet series of hydrogen lines is nothing but the KX-ray series of hydrogen.* Similarly, it is equally certain that the LX-ray series of hydrogen is the ordinary Balmer series in the visible region, the head of which is at $365\ \mu\mu$. In other words hydrogen's ordinary radiations are its X-rays and nothing more. There is also an M series for hydrogen discovered by Paschen in the ultra-red, which in itself would make it probable that there are series for all the elements of longer wave-length than the L series, and that the complicated optical series observed with metallic arcs are parts of these longer wave-length series. As a matter of fact an M series has been found for six of the elements of high atomic number. Thus the Moseley experiments have gone a long way toward solving the mystery of spectral lines. They reveal to us clearly and certainly the whole series of elements from hydrogen to uranium, all producing spectra of remarkable similarity, at least so far as the K and L radiations are concerned, but scattered regularly through the whole frequency region, from the ultra-violet, where the K lines for hydrogen are found, clear up to frequencies $(92)^2$ or 8464 times as high. There can scarcely be a doubt that this whole field will soon be open to our exploration. How brilliantly, then, have these recent studies justified the predictions of the spectroscopists that the key to atomic structure lay in

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[1] This assumption is in this csse inconsistent with the simple form of Bohr's theory, although that theory still gives the ratio of the radii of the first and second orbits 1 to 4.

the study of spectral lines. The prophets little dreamed, however, that the study of spectral lines meant the study of X-rays. But now, through this study, a sub-atomic world stands revealed to us in simpler form than could have been imagined. For the atoms are now seen to be, in their inner portions at least, remarkably similar structures, with central nuclei which are exact multiples of the positive electron, surrounded in each case by electronic orbits which have certainly, so far as the inner ones are concerned, practically the same relations in all the elements, the radii of all these orbits being inversely proportional to the central charge or atomic number.

So far nothing has been said about a quantum theory or a Bohr atom. The results have followed from the known properties of assumed circular electronic orbits combined with Moseley's experimental law, and supplemented by the single additional assumption that the observed frequencies of corresponding lines from different atoms are proportional to the orbital frequencies. If they suggest, however, that the experimental facts do not necessitate the quantum theory for their more complete interpretation, the consideration of the energy relations involved—these have been entirely ignored thus far—reveals at once the futility of that hope, or of that fear, according to the nature of your predilections with regard to theory of quanta. *For the experimental facts and the law of circular electronic orbits have limited the electrons to orbits of particular radii.* But the energy principle does not permit them to be so limited without a sudden or explosive loss of energy whenever the orbit is obliged to change. Suppose, for example, that a cathode ray strikes the atom and knocks out an electron from a particular orbit. When this or some other electron returns from infinity to this orbit, it must in this act adjust its energy to the only value which is consistent with this orbit and its characteristic frequency. Hence in the act of readjustment it must radiate a definite quantity of energy. Or again, suppose that the nucleus loses a beta ray through the radioactive process. Every electronic orbit must then adjust itself to the new value demanded by Moseley's law. But this it cannot do if its energy is conserved. The only way to permit it to do so is to let it radiate a definite amount of energy in the act of adjustment. This suggests that each emission of a beta ray by a radioactive substance must be accompanied by a whole series of characteristic gamma rays corresponding to each changed orbit. The emission of an alpha particle, on the other hand, would require an absorption rather than an emission of energy, since its egress diminishes rather than increases the nuclear charge. Perhaps this is why beta rays are always accompanied by gamma rays, while alpha rays are not so accompanied. This is, however, a speculation which does not immediately concern us here. The important conclusion, for the purposes of our present subject, is that Moseley's facts and unquestionable mechanics, combined with our two assumptions of circular orbits and radiation frequencies proportional in different atoms to corresponding orbital frequencies, lead inevitably to the explosive emission of energy in definite quantities accompanying orbital readjustments. And there is nothing particularly disturbing or radical about this conclusion either, for we have no basis for knowing anything about how an electron inside an atom emits its radiation. The act of orbital readjustment would be expected to send out ether waves. The only difficulty lies in the conception of the stable, non-radiating orbits between which the change occurs, and whether or not we can see how such orbits exist, the experimental evidence that they do so exist is now very strong, and it is to further evidence for their existence, since that is the main point to be established if this theory of atomic structure is to prevail, that I now wish to direct your attention.

I have already mentioned the facts of magnetism and of the Zeeman effect which support the orbital point of view. But the strongest evidence is found in the extraordinary success of the Bohr atom, which was devised before any of these Moseley relationships, which have forced us to the essential elements of the Bohr theory,[1] had been brought to light. Bohr, however, was guided solely by the known character of the line spectra of hydrogen and helium, together with the rapidly growing conviction, now dissented from, so far as I know, by no prominent theoretical physicist, that the act of emitting electromagnetic radiation by an electronic constituent of an atom must, under some circumstances, though not necessarily under all, be an explosive process. To show what is the character of this evidence, let us consider first what are the essential elements in the Bohr theory and second what have been the accomplishments of that theory. Bohr's experimental starting point is the Balmer series in hydrogen the frequencies in which are exactly given by

$$\nu = N\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \tag{4}$$

n_1 having always, for the lines in the visible region, the value 2, and n_2 taking in succession the values 3, 4, 5, etc. As previously noted, Paschen had already brought to light a series in the infra red in which n_1 was 3 and n_2 took the successive values 4, 5, 6, etc. Lyman's discovery, subsequent to the birth of the Bohr atom, of an ultra-violet series of hydrogen lines in which n_1 is 1 and n_2 takes the values 2, 3, 4, etc., is not to be regarded as a success of the Bohr atom, but merely as a proof of the power of the series relationships to predict the location of new spectral lines. To obtain an atomic model which will predict these series relationships for the simplest possible case of one single electron revolving around a positive nucleus, Bohr assumed

A. A series of non-radiating orbits governed by equation (1). This is the assumption of circular orbits governed by the laws which are known to hold inside as well as outside the atom.

B. Radiation taking place only when an electron jumps from one to another of these orbits, the amount radiated and its frequency being determined by $h\nu = A_1 - A_2$, h being Planck's constant and A_1 and A_2 the energies in the two stationary states.

This assumption gives no physical picture of the way in which the radiation takes place. *It merely specifies the energy relations which must be satisfied.*

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[1] N. Bohr., Phil. Mag., 26, 1, and 476, and 857, 1913. Also 29, 332, 1915; 30, 394, 1915.

The principle of conservation of energy obviously requires that the energy radiated be $A_1 - A_2$. Also this radiation must be assigned some frequency ν, and Bohr placed it proportional to the energy because of the Planck evidence that ether waves originating in an atom carry away from the atom an energy which is proportional to ν.

C. The various possible circular orbits for the case of a single electron rotating around a single positive nucleus to be determined by $T = \frac{1}{2}\tau hn$ in which τ is a whole number, n is the orbital frequency, and T is the kinetic energy of rotation. This condition was imposed by the experimentally determined relationship of the frequencies represented by the Balmer series.

It will be seen that, if circular electronic orbits exist at all, no one of these assumptions is in any way arbitrary. Each one of them is merely the statement of the existing *experimental* situation. The results derived from them must be correct if the original assumption of electronic orbits is sound. Now it is not at all surprising that A, B and C predict the sequence of frequencies found in the hydrogen series. They have been made on purpose to do it, *except for the numerical values of the constants involved.* It was this sequence which determined the form given to C. The evidence for the soundness of the conception of non-radiating orbits is to be looked for then, first *in the success of the constants, and second in the physical significance, if any, which attaches to assumption C.* If the constants come out right within the limits of experimental error, then the theory of non-radiating electronic orbits has been given the most critical imaginable of tests, especially if these constants are accurately determinable.

What are the facts? The constant of the Balmer series in hydrogen is known with the great precision attained in all wave-length determinations and has the value 3.290×10^{15}. From A, B and C it is given by the simplest algebra as

$$N = \frac{2\pi^2 e^2 E^2 m}{h^3} \tag{5}$$

I have recently redetermined e^1 with an estimated accuracy of one part in 1,000 and obtained again the value 4.774×10^{-10} which I published in 1913.[1] I have also determined "h" photoelectrically[2] with an error, in the case of sodium, of no more than .5 per cent., the value for sodium being 6.56×10^{-27}. The value found by Webster[1] by the method discovered by Duane and Hunt[2] is 6.53×10^{-27}. Taking the mean of these two results, viz.: $6{,}545 \times 10^{-27}$ as the most probable value, we get with the aid of Bucherer's value of e/m which is probably correct to one tenth per cent. $N = 3.294 \times 10^{15}$ *which agrees within a tenth per cent. with the observed value.* This agreement constitutes most extraordinary justification of the theory of non-radiating electronic orbits. It demonstrates that the behavior of the negative electron in the hydrogen atom is at least correctly described by the *equation* of a circular orbit. If this equa-

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[1] R. A. Millikan, Proc. Nat'l Acad., April 1917.

[2] R. A. Millikan, Phys. Rev., VII., 362, 1916.

tion can be obtained from some other physical condition than that of an actual orbit it is obviously incumbent upon those who so hold to show what that condition is. Until this is done it is justifiable to suppose that the equation of an orbit means an actual orbit.

Again, the radii of the stable orbits for hydrogen are given easily from Bohr's assumptions as

$$a = \frac{\tau^2 h^2}{4\pi^2 m e^4}.$$

In other words, since n is a whole number, the radii of these orbits bear the ratios 1, 4, 9, 16, 25. If normal hydrogen is assumed to be that in which the electron is on the inmost orbit, $2a$ the diameter of the normal hydrogen atom, comes out 1.1×10^{-8}. The best determination for the diameter of the hydrogen molecule yields 2.2×10^{-8}, in extraordinarily close agreement with the prediction from Bohr's theory. Further, the fact that normal hydrogen does not absorb at all the Balmer series lines which it emits is beautifully explained by the foregoing theory, since according to it normal hydrogen has no electrons in the orbits corresponding to the lines of the Balmer series. Again, the fact that hydrogen emits its characteristic radiations only when it is ionized favors the theory that the process of emission is a process of settling down to a normal condition through a series of possible intermediate states, and is therefore in line with the view that a change in orbit is necessary to the act of radiation. Similarly, the fact that in the stars there are 33 lines in the Balmer series, while in the laboratory we never get more than 12 is easily explicable from the Bohr theory, but no other theory has offered even a suggestion of an explanation. But while these qualitative successes of the Bohr atom are significant it is the foregoing numerical agreements which constitute the most compelling of evidence in favor of the single arbitrary assumption contained in Bohr's theory, viz.: the assumption of non-radiating electronic orbits.

Another triumph of the theory is that the assumption C, devised to fit a purely empirical situation, viz., the observed relations between the frequencies of the Balmer series is found to have a very simple and illuminating physical meaning, viz., the atomicity of angular momentum. Such relationships do not in general drop out of empirical formulae. When they do we usually see in them real interpretations of the formulae—not merely coincidences.

Again the success of a theory is often tested as much by its adaptability to the explanation of deviations from the behavior predicted by its most elementary form as by the exactness of the fit between calculated and observed results. The theory of electronic orbits has had remarkable successes of this sort. Thus it predicts, as can be seen from 4, 5 and 3, the relationship which we assumed, viz., that for corresponding lines (like values of n_1 and n_2 in 4) the orbital frequencies n are proportional to the observed frequencies ν, and similarly it predicts the Moseley law (2). But this latter relation, which is the only one of the two which can be directly tested, was found inexact, and it should be inexact when there is more than one electron is the atom, as is the case save for hydrogen atoms and for such helium atoms as have lost one negative charge, and that because of the way in which the electrons influence one another's fields. It will probably be found to break down completely for very light atoms like those of lithium. The more powerful the nucleus, however, and the closer to it the inner orbit the smaller should this effect be. Now precisely this result is observed. The Moseley law holds most accurately when tested for hydrogen and the elements of highest atomic number and much less accurately when tested for hydrogen and aluminum or magnesium. Similarly the ratio between the frequencies of the α and β lines of the K series approaches closer to the theoretical value (that for hydrogen) the higher the atomic number of the element.

Again, it is now well known that the α, β, and γ lines in the characteristic X-ray spectra are not single lines as required by the simple theory. Accordingly Sommerfeld[1] extended Bohr equations in the endeavor to account for this structure on the basis of ellipticity in some of the orbits, and Paschen[2] by measurements on the structure of the complex helium lines has obtained so extraordinary checks upon this theory that e/m comes out from his measurements to within a tenth per cent. of the accepted value.

A further prediction made by the theory and discovered as soon as looked for was the relation between the lines of two succeeding series of this sort.

$$\nu_{K\beta} - \nu_{K\alpha} = \nu_{L\alpha}$$

This should hold accurately from the energy relations between the orbits whether there be one or many electrons in the atoms. I have been able to find no case of its failure, though the data upon which it may be tested is now considerable. I have also recently pointed out[3] that it is equivalent to the well-known Rydberg-Schuster law[4] which has been found to hold quite generally among optical series. Finally the ionizing potential of hydrogen is given by Bohr's equations as 13.54 volts while experiment yields 11.5 volts. This discrepancy is no way prejudices the theory, but rather lends it support, for the computed value is for the hydrogen atom while the observed value relates to the hydrogen *molecule* which, in view of the repulsions between its two negative electrons might be expected to be ionized more easily. Similarly the computed value for helium which has lost one negative electron is 52.4 volts, but the neutral helium atom is found experimentally to be ionized at the much lower value 20.5 volts. That Bohr computed this latter value at 27 instead of 20.5 volts is not at all serious, since he had to make very particular assumptions to obtain this result.

If then the test of truth in a physical theory is large success both in the prediction of new relationships and in correctly and exactly accounting for old ones, the theory of non-radiating orbits is one of the well-established truths of

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[1] Annalen der Physik, 51, 1, 1916.

[2] Annalen der Physik, Oct., 1916.

[3] PHYS. REV., May, 1917, presented before Amer. Phys. Soc., Dec. 1, 1916.

[4] Baly's Spectroscopy, p. 488.

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PLATE I.
To face page 204.

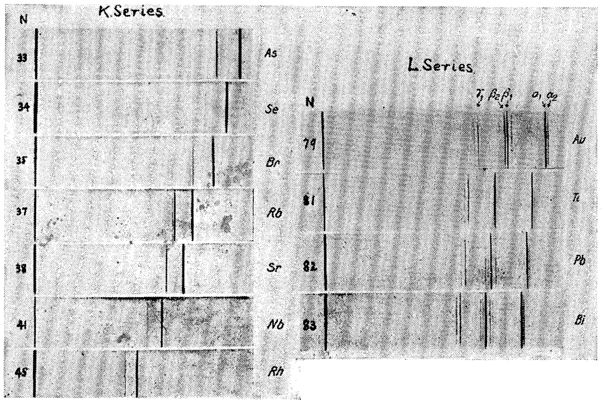

Fig. 1.

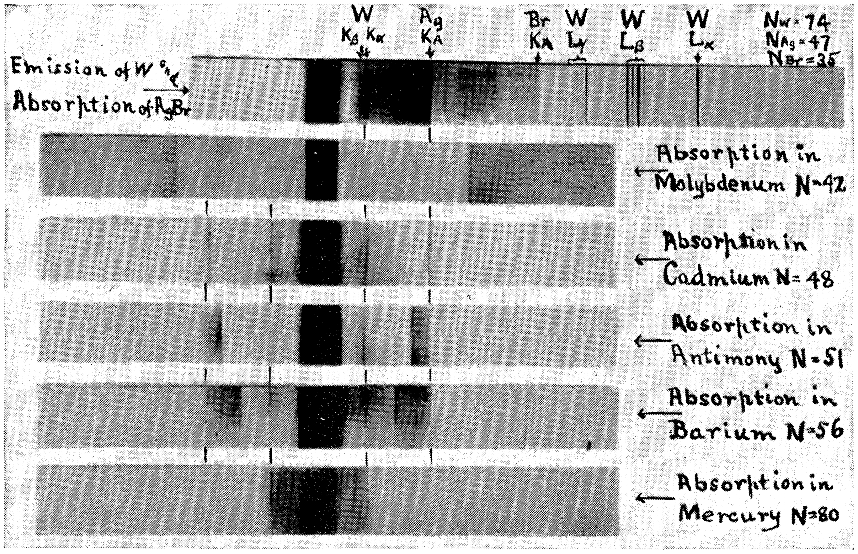

Fig. 2. Showing absorption (in K region) of general radiation of tungsten by various substances.

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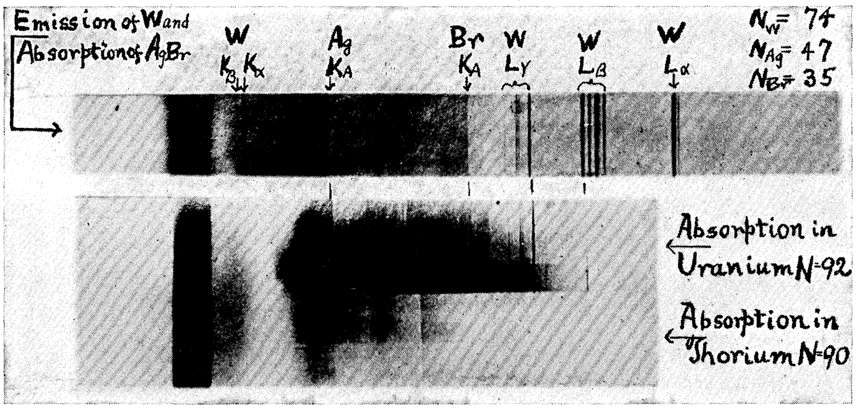

Fig. 3. Showing absorption (L region) by uranium and thorium.

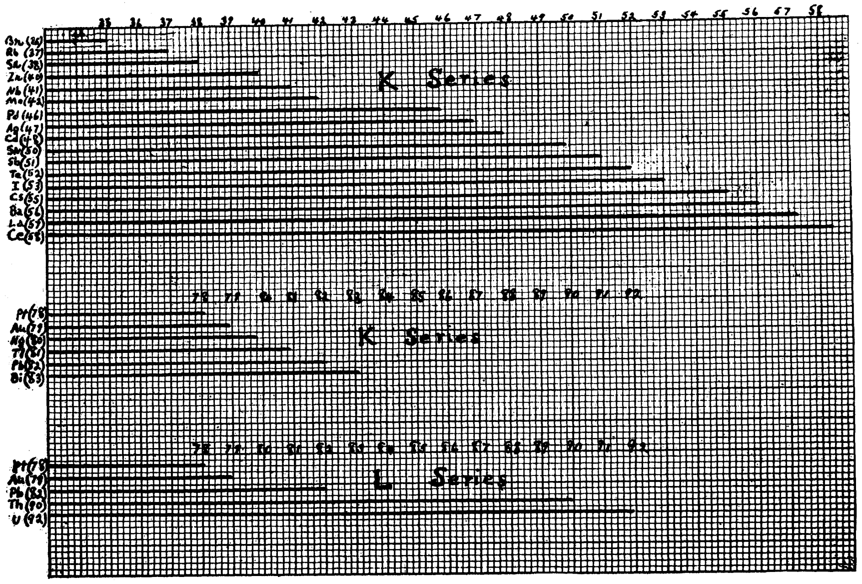

Fig. 4.

R. A. MILLIKAN.

modern physics. For the present at least it is truth, and no other theory of atomic structure need be considered until it has shown itself able to approach it in fertility. I know of no competitor which is as yet even in sight. I am well aware that the facts of organic chemistry seem to demand that the valence electrons be grouped in certain definite equilibrium positions about the periphery of the atom, and that at first sight this demand appears difficult to reconcile with the theory of electronic orbits. As yet, however, there is no necessary clash. Hydrogen and helium present no difficulties, since the former has but one valency, and the latter none. It is to these atoms alone that the unmodified Bohr theory applies, for it treats only the case of a single negative electron rotating about a positive nucleus. That the K radiations of the heavy elements are so accurately predictable from those of hydrogen indicates indeed that close to the nucleus of these elements there lie electrons to which the Bohr theory fairly accurately applies, but the radiations give us no information about the conditions or behaviors of the external electrons which have to do with the phenomena of valency, and we have investigated but little the radiating properties of the atoms which possess but few electrons. A further study of the behavior with respect to X-rays of the elements from lithium, atomic member 3, to magnesium, atomic number 11, may be expected to throw new light on this problem.

It has been objected too that the Bohr theory is not a radiation theory because it gives us no picture of the mechanism of the production of the frequency ν. This is true, and therein lies its strength, just as the strength of the 1st and 2nd laws of thermodynamics lies in the fact that they are true irrespective of a mechanism. The Bohr theory is a theory of atomic structure; it is not a theory of radiation, for it merely states what energy relations must exist when radiation, whatever its mechanism, takes place. As a theory of atomic structure, however, it is thus far a tremendous success. The radiation problem is still the most illusive and the most fascinating problem of modern physics. I hope to discuss it at a later time.

Amplification of the Photoelectric Current by the Audion.[1]

By Jakob Kunz.

The photoelectric cell is used already as a photometer for many scientific investigations, for instance in stellar photometry, in plant physiology, in researches on phosphorescence, on transmission, absorption, reflexion and radiation of light in various forms. It would find further applications, even for technical purposes, if the photoelectric current were larger. The writer has some time ago constructed a photoelectric relay, where the primary photoelectric current was increased considerably by a second electric field in the cell. However this relay has not been studied and developed sufficiently and it was

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

found that the audion, which has already so many important applications, amplifies the photoelectric current to a higher degree than the relay. The arrangement of the apparatus is given in the following figure, where P represents the photoelectric cell, G_1 and G_2 two galvanometers, B_1, B_2, B_3 batteries of 120, 20 and 8 volts; A the audion. The photoelectric cell was illuminated by an incandescent lamp of 2 candles at various distances from 30 to 170 cm. from the cell; the battery B_3 supplies the heating current of the filament of the audion. The galvanometer G_1 of the primary circuit was 32 times more sensitive than G_2 of the secondary circuit. The amplification of the photoelectric current by the audion depends largely on the temperature of the incandescent filament; the higher the temperature, the greater the secondary current, as will be seen from the following readings.

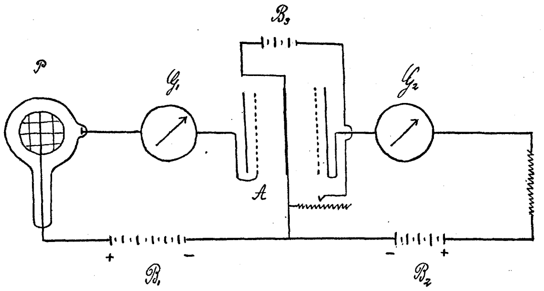

Fig. 1.

34, 5	102
34, 0	130
34	180
33	250
33	500

When the deflection of the secondary galvanometer was 500, the temperature of the filament was that of beginning white heat; the secondary deflection is 15 times larger than the primary, or the secondary current is 480 times larger than the primary photoelectric current. And it was easy by raising the temperature to amplify the photoelectric current 1000 times. With higher temperatures the secondary currents become less steady. A small variation of illumination produces a large variation in the secondary deflection. The voltage applied to the photoelectric cell connected with the audion can be raised above the point, where the glow in the cell sets in when independent of the audion.

Laboratory of Physics,
University of Illinois,
Urbana, Illinois, April, 1917.

High Vacuum Spectra from the Impact of Cathode Rays.[1]

By Louis Thompson.

E. P. LEWIS, A. S. King and others have described experiments on the production of metallic spectra by cathodo-luminescence.

R. H. Goddard has devised a means[2] of increasing the light in one section of the tube, due to the impact of cathode rays, thus making it possible to work with much higher vacua. The method consists essentially in the use of a magnetic field applied parallel to the axis of two small negatively charged plates, which also serve as cathodes. The combined effect of the electric and magnetic fields is to cause the electrons to assume a closed helical path in the very limited region between these plates. This results in a small but bright cylinder of light (between *A*, *B* in the figure) even at extremely low pressures, where without the field, the illumination would be so faint that satisfactory exposures would be impossible.

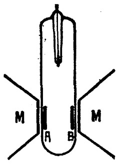

Fig. 1.

In addition to the advantage of having sufficient light for comparatively short exposures at high vacua, there are no temperature effects and the pressures are definitely known, especially when working with substances which are normally gaseous. The use of capillary tubes for high vacuum work, while possible, gives spectra which depend largely upon temperature and current density.

At high vacua and with temperature effects absent, simple spectra are likely to result. A recent paper by Mallik and Das[3] emphasizes this point.

A number of substances are being examined. The first exposures were made with mercury vapor and tellurium, the mercury at pressures down to that corresponding to a three-inch parallel spark.

Compared with the ordinary vacuum tube discharge the spectrum is simple and the relative intensities are noticeably different. The important series lines are prominent in the visible region. In the ultra-violet only 3131, 3125 and 2536 are strong. 2536 is the brightest line in the spectrum and seems to become relatively stronger as the vacuum increases.

Experiments are being made with hydrogen in an attempt to determine whether at high vacua and with the increased illumination, the extended Balmer series, which has only appeared in certain stellar spectra, can be found in a discharge tube.

The magnetic field used in the work is not strong enough to produce a detectable effect on the spectrum.

Clark University,
Worcester, Mass.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

[2] Described in U. S. Patent 1,137,964.

[3] Phil. Mag., March, 1917.

A PROPOSED METHOD FOR THE PHOTOMETRY OF LIGHTS OF DIFFERENT COLORS. —III.[1]

BY IRWIN G. PRIEST.

TWO papers under the above title[2] have already been communicated to the Physical Society by the author. The purposes of the present paper are:

1. To present some data on spectral distribution in order to further elucidate and substantiate the method.

2. To publish, for reference, a table of numerical values of the method factor mentioned in the first paper and explicitly defined in the second paper as

$$R = \frac{\int_{400}^{740} VE \sin^2 (\phi - \alpha) d\lambda}{\int_{400}^{740} VE d\lambda}.$$

3. To note some advantages and possible applications of the method.

1. *The Relative Spectral Distribution of Lights Color-matched by the Proposed Method.*—The essential feature of the method is that light from a comparison source of known spectral distribution is modified by passage through a quartz plate between nicol prisms so as to match the color of the source being tested. The quartz plate and the nicols constitute, in effect, a " color screen " of readily adjustable spectral transmission, the relative transmission for any wave-length being given by[3]

$$\sin^2 (\phi - \alpha).$$

Typical curves showing values of this expression as a continuous function of λ for various values of ϕ (" spectral transmission curves ") are shown in Fig. 1.

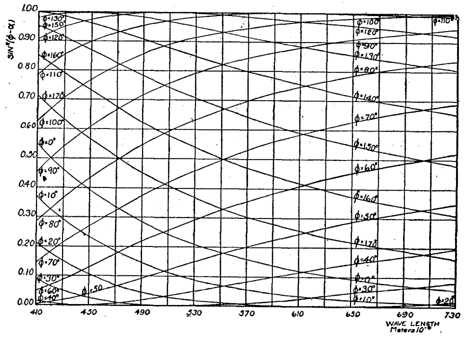

Fig. 1.

Relative transmission of a system composed of a 1 mm. plate of quartz between nicol prisms. α = rotation of plane of polarization by the quartz plate (a function of wave-length); ϕ = rotation of analyzing nicol from position for extinction (quartz removed). (Both α and ϕ are measured clockwise from observer's position).

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[1] Abstract of a paper presented (by title) at the Washington meeting of the Physical Society, April, 1917.

[2] Am. Phys. Soc., Wash., April, 1915; PHYS. REV. (2), 6, 64. Am. Phys. Soc., N. Y., Dec., 1916; PHYS. REV. (2), 9, 341 (for erratum see p. 580).

[3] For definition of symbols see PHYS. REV., 9, 343.

In the previous papers it was merely stated that a sensibly perfect color-match had been obtained. Fig. 2 in the present paper compares the spectral distribution in light from the carbon lamp at 4 w.p.m.h.c.[1] with the spectral distribution of the light indirectly made to color-match it by the proposed method in the determination of relative candlepowers reported in the second paper cited above.

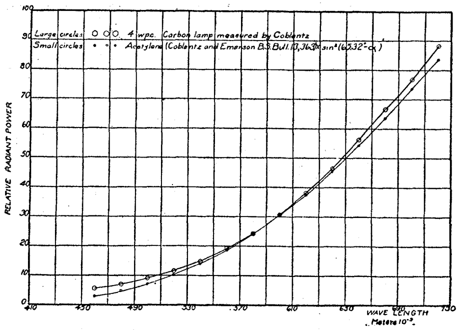

Fig. 2.

Two spectral radiation distributions giving an empirical color match in the relative candlepower determinations reported in paper No. II. of this series.

Fig. 3 shows the theoretical possibility of color-matching, by this method, the light from a gas-filled tungsten lamp. This does not represent an empirically determined color-match, but merely compares the spectral distribution of the gas filled lamp,[2] with that of acetylene as modified by the quartz-nicol system, the thickness of quartz and the angle ϕ having been theoretically chosen to give the desired spectral transmission. The exact efficiency of the gas-filled lamp in this case is not known, but was probably about 15 lumens per watt.

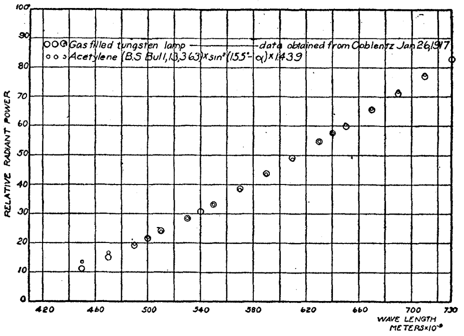

Fig. 3.

Illustration of the matching of a given spectral radiation distribution by a distribution computed by the rotatory dispersion method.

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[1] Determined radiometrically by W. W. Coblentz and communicated to the author.

[2] Radiometrically determined by W. W. Coblentz.

Fig. 4 compares the radiometrically determined spectral distribution[1] of a gas-filled lamp at 15.6 lumens per watt[2] with the spectral distribution of the light empirically found to color-match it by this method, the match being made by the author using the Arons Chromoscope.[3]

The dotted and dashed curves in Fig. 5 show the spectral distributions of lights found, by this method, to color-match a gas-filled lamp at 22.3 lumens per watt and at 22.0 lumens per watt.[4]

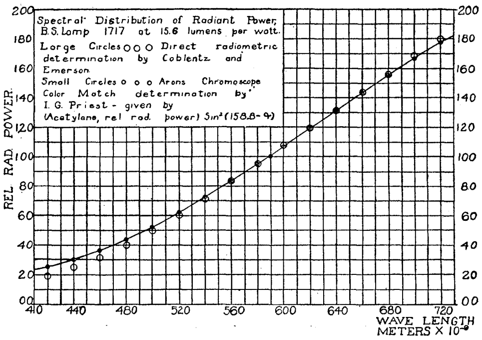

Fig. 4.

The solid curve from Luckiesh, " Color," page 21, is given for comparison.

When it is considered that the efficiency of a gas-filled lamp can not be taken as a reliable specification of its color, and that the photometry of these lamps is a matter of considerable uncertainty, this discrepancy appears hardly greater than might be expected. It will be noted that the agreement of curves in Fig. 4, where the data are strictly comparable (pertaining to the *same lamp*), is considerably better.

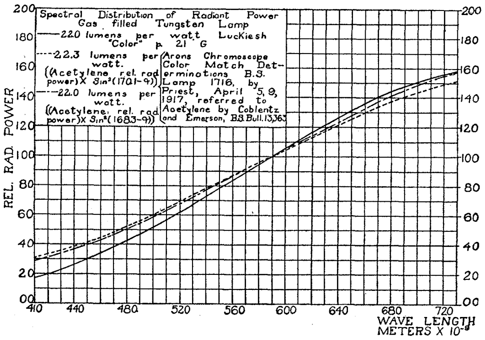

Fig. 5.

It will be understood, of course, that the author has no idea of here testing

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[1] By Coblentz and Emerson, Bureau of Standards.

[2] By Crittenden and Taylor, Bureau of Standards.

[3] Ann. der Phys. 39, 545, 1912.

[4] By Crittenden and Taylor, Bureau of Standards.

the accuracy of spectral distributions determined radiometrically or spectrophotometrically. The purpose of presenting this data is merely to show how closely this method of empiric *color-matching* does, in fact, give a spectral match. Those familiar with the subject will readily see that this method of color-matching gives better spectral matches than are obtained by colored glasses, which have been used for the same purpose.

A given source may be color-matched using quartz plates of different thicknesses, each plate, of course, requiring a different value of ϕ to give the color-match. For example, with acetylene as a comparison source, a gas-filled tungsten lamp at 12.3 lumens per watt may be color-matched with the following thicknesses of quartz and corresponding values of ϕ:

Thickness, mm.	ϕ, degrees.
0.50	162.4
.75	157.9
1.00	154.6

The spectral distribution curves from these three sets of data are much the same; but have, indeed, slightly different curvatures, so that one of them may match the true spectral distribution of the lamp better than the others. Further consideration is being given to the choice of the quartz thickness best suited to give a very close spectral match where that is desired, but 1.000 mm. appears very satisfactory for the photometry of the incandescent lamps now in use (carbon, vacuum tungsten and gas-filled tungsten).

2. *Table of the Method Factor.*—In the previous papers, it was pointed out that computations in terms of this method, while apparently quite involved, could be made very simple by the use of permanent reference tables of the "method factor," R. The labor of preparing such a table, based on the most recent and reliable data for "visibility" and "spectral distribution of radiant power" has now been completed,[1] and the table is presented herewith in the hope it will be useful to any who care to investigate or use the method.

For illustrative purposes, the form of the function R is shown in Fig. 6.

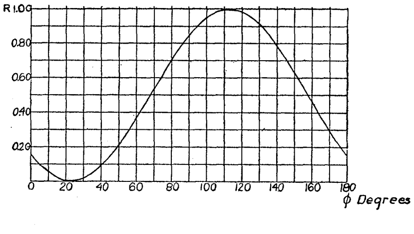

Fig. 6.

All steps in the preparation of this table have been carefully checked and the author trusts that it will be found reliable. He hopes however that some one may find interest enough in the subject to check this table entirely independently assuming nothing except the same fundamental data for E, V and α, and the arbitrary values for ϕ.

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[1] Graphic integration and computation by H. J. McNicholas, J. T. Filgate and H. E. Cole.

As the subject develops, it may be later found desirable to know R for some particular values of ϕ more accurately than it is given in the present table. The author hopes to undertake such determinations later. Before this method can be generally adopted, it will of course be necessary for those interested and in authority to agree upon standard values for V and E as used in the formula for R. Instead of taking E for acetylene, it may be best to refer directly to a black body at some specified temperature.

3. *Notes on Advantages and Applications of the Method.*—(*a*) It is evident from the data presented above that this method is superior to the color glass method from the point of view of obtaining a close spectral match as well as a "color-match."

(*b*) Selective transmission obtained and controlled in this way is universally reproducible and subject to fine adjustment, while that obtained by glasses, is, in general, neither.

(*c*) Spectral transmission determined by this method is more reliable than that of a glass determined by means of the spectrophotometer; and, by use of permanent tables, may be more readily obtained.

(*d*) The data obtained in determining candlepower may be readily translated into a significant specification of the color of the light being tested.

(*e*) The method is applicable to the production of "artificial daylight" from tungsten lamps, giving a much better spectral match than has been obtained by the use of blue glass. The author is preparing a separate paper on this subject.

(*f*) At the request of Dr. N. E. Dorsey, application of this method to determine the brightness of very faint greenish fluorescent screens is being attempted. A satisfactory color-match has been found.

(*g*) It is obvious that the photometer described in the preceding paper in this series, may be used as a pyrometer, and might be calibrated to read temperatures directly, but the author ventures no opinion as to its value for this purpose.

NATIONAL BUREAU OF STANDARDS,
April 17, 1917.

THE METHOD FACTOR R AS A FUNCTION OF ϕ

Computed by H. J. McNicholas, J. T. Filgate and H. E. Cole, Dec., 1916–Jan., 1917.

$$R = \frac{\int_{400}^{740} VE \sin^2 (\phi - \alpha) d\lambda}{\int_{400}^{740} VE \, d\lambda}$$

assuming the following functions of λ:

V = Relative Visibility, Coblentz and Emerson, Phys. Rev. *9*, 88.

E = Relative Radiant Power Acetylene Flame, Coblentz & Emerson, B. S. Bull 13, 363.

α = Specific Rotation of Quartz (Degrees/mm), Landolt, Op. Dreh. 2nd Ed., p. 128; and Landolt Börnstein Tables, 1913 Ed., p. 1062.

(This table was constructed by graphic interpolation from values determined by graphic integration at intervals of 5°. The values tabulated are means of two independent interpolations. They are considered reliable to about 0.001.

ϕ.	R.	ϕ.	R.	ϕ.	R.	ϕ.	R.
1°	0.1425	46°	0.1545	91°	0.856	136°	0.844
2	132	47	1675	92	867	137	831
3	121	48	1805	93	879	138	818
4	1105	49	194	94	889	139	805
5	1005	50	209	95	899	140	791
6	091	51	222	96	9095	141	777
7	081	52	2365	97	9195	142	7615
8	072	53	251	98	9285	143	747
9	064	54	266	99	938	144	7315
10	056	55	2815	100	946	145	716
11	048	56	297	101	954	146	700
12	041	57	3135	102	9605	147	6845
13	034	58	330	103	9675	148	668
14	028	59	347	104	973	149	652
15	023	60	364	105	978	150	636
16	018	61	3805	106	983	151	620
17	014	62	397	107	9865	152	6035
18	011	63	4135	108	990	153	586
19	008	64	4305	109	993	154	570
20	006	65	447	110	995	155	552
21	004	66	4645	111	9955	156	5355
22	004	67	482	112	9955	157	518
23	0035	68	4995	113	995	158	5005
24	004	69	5165	114	994	159	4825
25	005	70	534	115	9925	160	4645
26	006	71	551	116	991	161	448
27	008	72	5675	117	9885	162	430
28	0115	73	585	118	9865	163	413
29	0145	74	6015	119	984	164	3955
30	5185.	75	6185	120	980	165	378
31	0235	76	6355	121	9765	166	361
32	029	77	652	122	9715	167	345
33	0355	78	6685	123	9665	168	329
34	042	79	685	124	960	169	3125
35	049	80	7015	125	953	170	2965
36	056	81	717	126	9465	171	2805
37	064	82	733	127	9385	172	266
38	072	83	7485	128	9305	173	2505
39	081	84	763	129	922	174	236
40	090	85	7775	130	9125	175	222
41	099	86	7915	131	902	176	2075
42	109	87	8055	132	891	177	1935
43	1195	88	819	133	8805	178	1805
44	1305	89	8325	134	8685	179	167
45	142	90	8445	135	856	180	1545

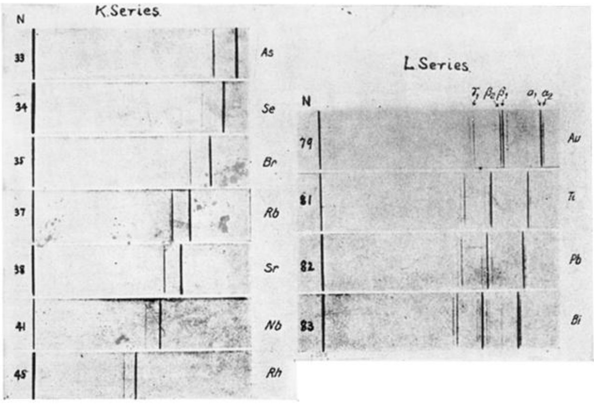


Fig. 1.

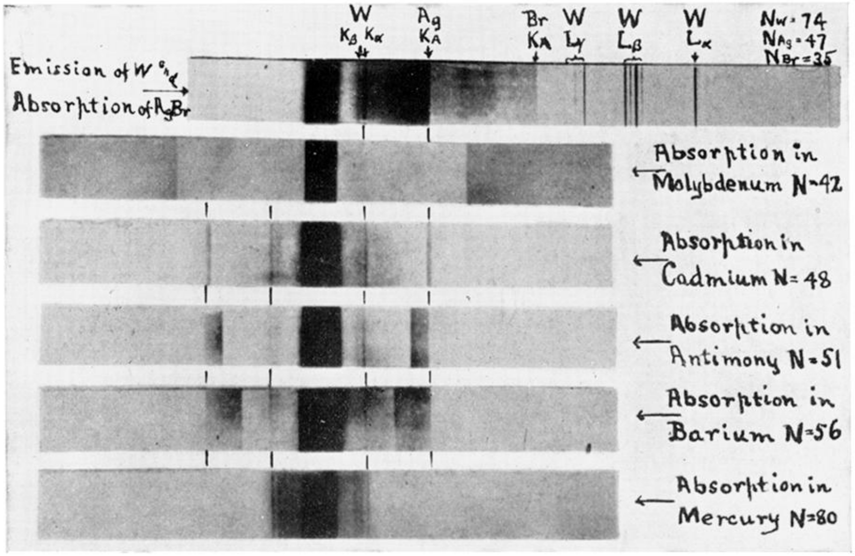
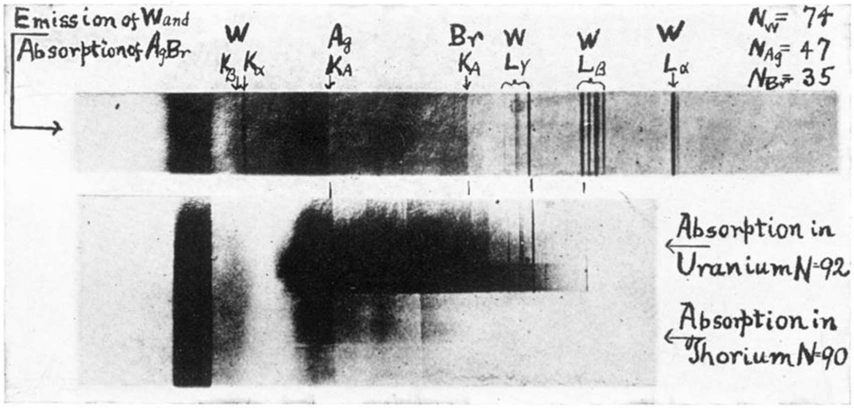


Fig. 2. Showing absorption (in K region) of general radiation of tungsten by various substances.



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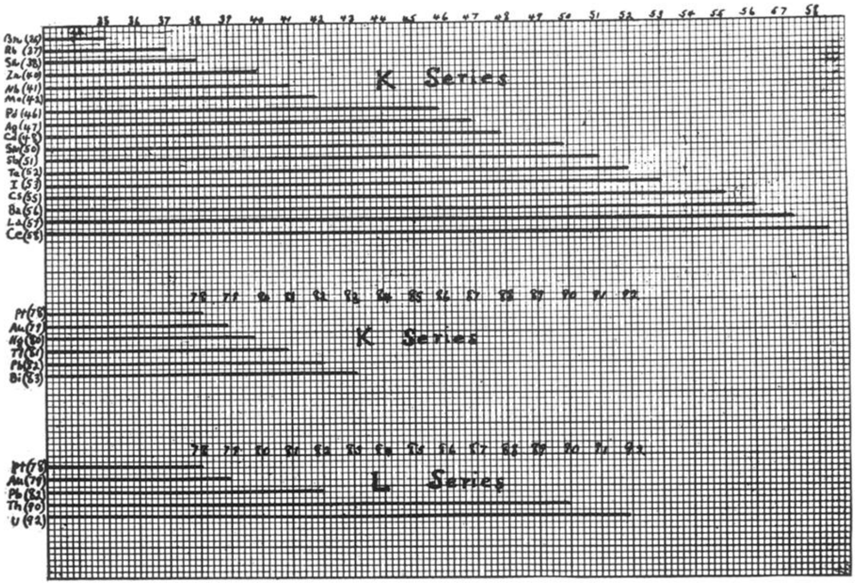

Fig. 4.