A SYSTEMATIC STUDY OF LINEAR AND NON-LINEAR RESONATORS FOR SHORT ELECTRIC WAVES.

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Introduction.

IN the preceding paper the so-called "multiple reflection method" for investigating the radiation from screens of resonators has been discussed. The results and advantages of this method have been given there. It was shown that multiple reflection gave a radiation which was little damped and contains a single wave-length. With this kind of radiation we have determined the relation of the dimensions and the distribution of the resonators to the wave-length, damping, and amount of energy radiated from a screen. The question as to how wide the resonator may be and still be considered linear has also been investigated, in as much as the term "linear resonator" has been used rather loosely by other investigators.

The apparatus described in the first paper was used again in this investigation. There was a primary oscillating system (*source*), a reflecting system, and a receiving system. The oscillating system, or primary source of energy, consisted of a Righi vibrator placed at the focus of a spherical parabolic mirror. The reflecting system was the screens of tin-foil resonators to be studied together with a Boltzmann mirror for measuring the wave-lengths. The receiving system had a non-selective receiver of the Klemenčič type at the focus of a parabolic mirror similar to that used in the oscillating system. There was also a non-selective check receiver, the use of which was explained in the previous paper. The dimensions and details of the apparatus, as well as the arrangement of the parts, are given at length in the same place. Throughout the present investigation the apparatus was kept as nearly as possibly the same. It was found best to keep the period of the primary source as nearly as possible the same as the natural period of the resonators under investigation. This was accomplished by changing the vibrator from time to time.

A change in the dimensions of the resonators must be considered as due to a variation of the length, width or thickness: and by a change in the distribution is to be understood a change in the distance between consecutive resonators measured parallel to the electric component or a change in the distance measured perpendicular to this. The thickness of all the resonators was kept small and constant. In the discussion which follows, the dimension of a resonator measured parallel to the electric component of the oscillation in the incident energy is called the length, the dimension perpendicular to this is called the width. In general, the length was greater than the width but in some cases the width was

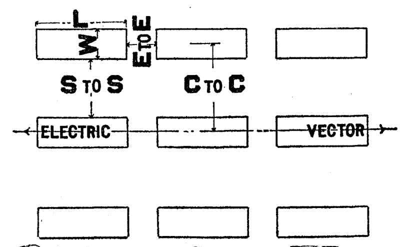

Fig. 1.

C to *C*, center-to-center distance; *S* to *S*, side-to-side distance; *E* to *E*, end-to-end distance; *W*, width; *L*, length.

several times as great as the length. Fig. 1 will make clear the meaning of the terms to be used.

The length of a resonator is the most important factor in determining its natural frequency. It seems well established that the wave-length reradiated is directly proportional to the length of the linear resonator, if the separation be kept in a constant ratio to the length of the resonator. The relation may be expressed by an equation of the form $\lambda = KL$, where K is a constant whose value depends on other factors than the length. The value of this constant given by theoretical and experimental investigators[1] seems to lie between 2 and 2.6. The theoretical values apply to the case in which there is but one resonator in the field. The primary object of this investigation was to find the value of K for any distribution of resonators for which the corresponding energy is sufficiently large to permit of the application of the present apparatus and method.

The results of this investigation are presented under three headings, viz, the effect on the period of oscillation of the resonators brought about by a change in

I. The distance between the resonators measured *parallel* to the electric component in the energy,

[1] MacDonald, Electric Waves, p. 111; Poincare, Les Oscillations Electric; M. Abraham, Wied. Ann., 1898, 66, p. 435; Blake and Fountain, PHYS. REV., 1906, Vol. XXIII., p. 276; Webb and Woodman, PHYS. REV., 1909, Vol. XXIX., p. 131; Ives, PHYS. REV., 1910, Vol. XXX., p. 199.

II. The distance between the resonators measured *perpendicular* to the electric component in the energy,

III. The *width* of the resonator.

I. The Effect of a Change in the Distance Between the Resonators Measured Parallel to the Electric Oscillation.

It was not expected that the distance between the ends of the resonators (*E* to *E*, Fig. 1) would affect their period of oscillation. This proved to be true for linear resonators. There is but little mutual capacity effect for such small surfaces, and there is no interlinking of the field of one with the circuit of the other, hence no mutual induction effect. Even with very wide resonators when the distance between the ends of the resonators was changed experiments showed that there was no effect on the period of oscillation of the resonator.

For the present "linear resonators" shall be considered as those having a width not greater than *one tenth* of the length. Later it will appear that a "linear resonator" may have a width as great as one fourth of the length. Table I. shows the results due to end-to-end variations. Using linear resonators, this end to end distance between resonators was varied from 4 mm. to 22 mm. The length of the resonators was changed from 23 mm. to 55 mm. The changes in wave-lengths noted were those due to changes in length of the resonators and changes in the distance between the resonators in the direction perpendicular to the electric oscillation. The fourth column of the table (Measured Wave-lengths) gives the wave-lengths measured by the Boltzmann mirror for the different screens of resonators; the fifth column gives the wave-length calculated for screens of resonators having the same dimensions and center-to-center distance, but having an end-to-end distance of 10 mm. How these calculated wave-lengths were obtained will appear in the next section, which relates to the effect of varying the center-to-center distance. The method of measuring the wave-lengths is accurate to about two per cent.; this is about the maximum variation of any measured wave-length from that calculated for the similar screen with a different end-to-end distance. The results in Table I. must be considered for each screen separately, for the separation center-to-center was not the same for the different cases, and screens having one separation cannot be compared with those having some other separation. With the resonators 226 and 590 mm. wide only one row of resonators was put on the screens, hence the different screens may be compared directly.

The fact that there was no end-to-end effect for the linear resonators

TABLE I.

Width, Mm.	Length, Mm.	End-to-end Distance, Mm.	Wave-length.	
			Measured, Mm.	Calculated for E to E 10 Mm., Mm.
2	23	22	58	57
2	36	10	95	96
2	36	15	97	96
2	50	4	113	113
2	55	5	110	113
3	36	10	83.5	85
3	36	15	95	97
18	36	10	91	91
18	36	20	81	81
18	36	15	100	97
226	36	10	72.3	
226	36	56	72	
590	50	43	110	
590	50	15	112	
590	50	20	110	

does not require that there be no such effect for the wide resonators. It was, however, demonstrated that there was no effect of this kind even for the very wide resonators. As appears in Table I., resonators varying in width from 18 mm. to 590 mm. (that is, from one half to twelve times the length) gave little end-to-end effect. With resonators 18 mm. wide there was no change in the wave-length when this end-to-end distance was changed from 10 mm. to 20 mm.; resonators 226 mm. wide showed no change in wave-length for a variation of this distance from 10 mm. to 56 mm. The end-to-end distance was changed from 15 to 43 mm. with the very wide resonators (590 mm. wide), the change in wave-length was less than two per cent.

This work on the effect of a change in the distance between the resonators measured parallel to the electric oscillation confirms the work of Blake and Fountain[1] and extends it to apply to the case of non-linear resonators. We find then that there is no effect on the wave-length of the radiation from a screen of resonators when the resonators are moved apart in the direction of the electric component of the oscillation.

II. The Effect of Varying the Distance Between the Resonators Measured Perpendicular to the Electric Component of the Oscillation in the Resonators.

A change in the distance between adjoining resonators measured perpendicular to the electric oscillation produces a change in the wave-

[1] Blake and Fountain, PHYS. REV., 1906, Vol. XXIII., p. 268.

length of the energy radiated by a set of resonators of given length. Table II. and Fig. 2, *A*, give the results of this investigation on the effect of this change of distance for the linear resonators (resonators 36 mm. long). The curve represents the relation between the wave-length and the center-to-center separation for these linear resonators. There is a minimum value for the wave-length when the separation has the value of 9 mm., *i. e.*, the distance between the resonators is 0.4 of the length of the resonators. From this point the curve is practically a straight line, the wave-length increasing as the separation is increased. This straight line relation cannot hold indefinitely, but apperas to hold past the point for which the separation has the value of 90 mm. or 2.5 times the length of the resonators. The separation represented by this value of 90 mm. is the greatest for which the multiple reflection methods could be used. For separations greater than this the energy became very small because of the few resonators in the field, and the method was then unreliable.

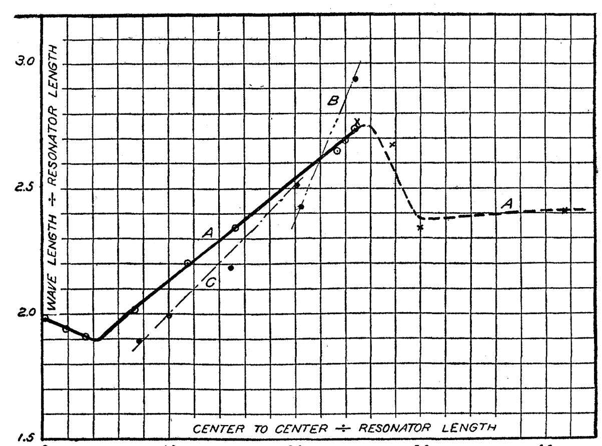

Fig. 2.

⊙, points taken by "multiple reflection"; ×, points taken by tuning method; ●, non-linear resonators.

The abscissæ of the curve as it is plotted in Fig. 2, *A*, were taken as the ratios of the distance between the resonators to the length of the resonator, and the ordinates as the ratios of the wave-length to the resonator length. These coördinates were chosen in order to make the curve more general in its application. With these coördinates the curve can be applied to any resonator system regardless of the length of the resonators. For with any two sets of resonators both variables are divided by the same factor, and the shape of the curve is maintained.

TABLE II.

C to C Distance, Mm.	Wave-length Measured, Mm.	$\frac{\text{Wave-length}}{\text{Reso. Length}}$.	$\frac{C \text{ to } C}{\text{Reso. Length}}$.	Wave-length of Source, Mm.
7	69	1.94	0.194	76
12	68	1.91	0.333	76
26	73	2.02	0.723	76
42	79.5	2.20	1.166	76
56	84.6	2.35	1.555	76
82	95	2.63	2.380	76
86.5	99	2.69	2.400	87
90	97	2.72	2.500	87
	All resonators 2 mm. wide and 36 mm. long.			

The data contained in Table II. were obtained by using the same linear resonators throughout, keeping the same distance between the ends of the resonators, and varying only the distance between the resonators in the direction perpendicular to the electric oscillation. As will be seen from the table, this distance was changed by successive small amounts from 7 mm. to 90 mm. In all cases two reflections were used and the source was kept fairly well tuned to the screens. Under these conditions, as shown in the preceding paper, the character of the resulting radiation depends almost entirely upon the resonators and is practically independent of the source.

For separations greater than 90 mm. or 2.5 times the resonator length, a method of tuning the receiver to the radiation from the screens had to be introduced. This method (described in the preceding paper) is open to the same objections[1] as the tuning methods used by other investigators. In any tuning method of wave-length measurement, the maximum energy may not and very probably is not received when the tuning is exact, *i. e.*, when the system has the same period as the incident energy, There is in all cases a decrease in the energy due to the decrease in the size of the resonator or receiver, as the case may be. This will tend to shift the length taken as the tuning length toward the large values and give an incorrect value for the wave-length. This objection may be brought against that part of the curve in Fig. 2, *A*, which lies beyond the abscissa 2.5. It may also be brought against the work of all investigators who have used a tuning method for studying resonators of this type.

Although the method of study is open to the above objections, it was adopted in the present investigations since it was very desirable to carry the curve beyond the value of 2.5 for the abscissa. From the part of the curve beyond this point it is possible to estimate the value of the

[1] Webb and Woodman, PHYS. REV., Aug., 1909, Vol. XXIX.

ordinate of the curve when the resonators are separated to such a distance that they no longer influence the period of one another. The value of the constant K connecting the resonator length and the wave-length when there is only one resonator in the field would thus be found. The exact shape of the curve beyond the point for which the abscissa is 2.5 has not been accurately determined. However, it appears to turn at about this point and to approach the ordinate having the value of about 2.4 as its value when the resonators no longer affect each other. This then would indicate the ratio of the wave-length to the resonator length when there is but one resonator in the field. The exact value of K is still in doubt, but the most probable value does not agree with the theoretical value given by Abraham.[1] The value does agree very well with that obtained by other investigators. For a radiating system the ratio of the wave-length to the resonator length seems to be greater than two.[2]

Table III.

C to C Dist., Mm.	Wave-length Measured, Mm.	Wave-length Calculated, Mm.	$\frac{C \text{ to } C}{\text{Reso. Length}}$	Length of Resonator, Mm.	E to E Dist., Mm.	Wave-length of Source, Mm.
42	58	57	1.825	23	22	72
42	58.5	57	1.825	23	22	76
42	91	90	1.000	42	10	87
42	94	90	1.000	42	10	118
67	113	113	1.340	50	4	87
42	111	113	0.722	55	5	76
84	116	120	1.400	60	10	118
42	119	122	0.700	60	10	118

As a test of the accuracy of the curve showing the relation of the separation to the wave-length emitted, a number of screens were made and the corresponding wave-lengths measured and compared with those calculated from the curve. The results are shown in Table III. In one column are given the measured values of the wave-lengths and in a parallel column the calculated values. Screens of resonators varying in length from 23 mm. to 60 mm. and having a C to C separation varying from 40 to 84 mm. were tested. The distance between the ends of the resonators are also given, but as shown in the last section this factor does not affect the wave-length. Linear resonators only were used, the width being 2 mm. in all cases. A comparison of the two columns mentioned above shows that in no case is the variation of the calculated value for the wave-length from the measured value greater than about 3 per cent. over a range of wave-lengths varying from 58 mm. to 119 mm.

[1] M. Abraham, Wied. Ann. d. Phys., 1898, 66, p. 435.

[2] Pierce's Principles of Wireless Telegraphy, p. 116.

Not only does this curve represent the results for the apparatus used in the present case, but the results of other investigators using other forms of apparatus agree very well with this curve. Aschkinass and Schaefer,[1] in 1901, published the results of some work done to determine the dielectric constants of certain liquids. Their investigation was made by studying the effect of the liquid on the period of resonators of the same type as ours when immersed in the liquid. The resonators had the dimensions and distribution given below:

Length of resonator..45 mm.
Separation perpendicular to length............................27 mm.
Width of resonators......................................."linear"
Wave-length in tune with resonators *in air*....................90 mm.

For a screen of this description the curve Fig. 2, *A*, calls for a wave-length of 90 mm.

Woodman and Webb[2] found that resonators 29 mm. long, with a separation of 30 mm., were in tune with a source having a "wave-length of 62 to 64 mm." The curve shows that a screen with resonators of these dimensions should have a natural frequency corresponding to a wave-length of 62.5 mm. The wave-lengths given by the various observers mentioned above were measured by different methods, and differing from that used in obtaining the curve, so that the agreement between their results and the calculated values cannot in any way be due to a similarity of apparatus or methods. Hence it is evident that the curve given in Fig. 2, *A*, which represents the effect of the separation on the wave-length expresses very closely the correct relation between these two quantities.

Blake and Fountain[3] in a paper relating to linear resonators, give the following data for three of their screens:

No.	Resonator Length, Mm.	Separation, Mm.
1	53	30
2	49	60
3	44	100

The curve obtained in the present investigation gives the following values for the wave-lengths from these screens:

No.	Wave-length.
1	104
2	109
3	115

[1] Aschkinass and Schaefer, Ann. d. Phys. 1901, 5, p. 489.
[2] Woodman and Webb, PHYS. REV., Vol. XXX., 1910, p. 575.
[3] Blake and Fountain, PHYS. REV., Oct., 1906, Vol. XXIII.

The investigators state that these screens were in tune with a vibrator having a wave-length of 99 mm.[1] Our study of their interference curve for the wave-length of their source seems to indicate that the wave-length may have been about 108 mm. Webb and Woodman[2] have shown that the tuning method is not very reliable for wave-length measurements and that the tuned receiver is unsatisfactory for this kind of work. The results given for these screens do not agree as well with the calculated results from the curve as those of the other investigators mentioned. But the errors to be expected from the tuning method are more than sufficient to account for the discrepancy.

In the same paper (page 268) Blake and Fountain have given a curve representing the relation of the separation to the wave-length for the screen. Their curve does not seem to fit the facts as found in the present investigation. The discrepancy is one of amount and not of kind.

The curve, Fig. 2, *A*, applies to linear resonators only. If the multiple reflection method is to be of value, we must be able to determine beforehand what the frequency of the resonators will be on any given screen. Hence the effect of the separation on the wave-length for the *non-linear* resonators must also be known. The effects for square resonators (36 mm. long and 36 mm. wide) and for resonators 36 mm. long and 18 mm. wide are shown in Fig. 2, *B*, and 2, *C*. It is seen that the curves are similar to that for the linear resonators but have a greater slope. Hence for wide resonators the wave-length increases by a greater amount for the same increase in separation than for the linear resonators. For the intermediate widths the slope of the curve is also intermediate.

When the wide resonators are used a question arises as to how this separation is to be measured. Shall it be taken as the distance between the adjacent edges of the resonators or shall it be taken as the distance between corresponding points in the resonators, as for example between the centers (*C* to *C*)? A study of this point shows that the results are more simply correlated if the separation be taken as the distance between the centers of the resonators. The reason for this conclusion will be taken up more fully in the next section.

The curves in Fig. 2 were used for determining the length and distribution of resonators on screens to give desired wave-lengths. The length chosen depended on the energy required, for the energy reradiated decreased with a decrease in the length of the resonator as well as with the separation. A very satisfactory condition was found when the length of the resonator was approximately one half of the wave-length desired.

[1] Blake and Fountain, Phys. Rev., 1906, Vol. XXIII., pp. 263–4.

[2] Webb and Woodman, l. c., p. 123.

Having chosen the length of the resonator the exact distribution to be used was determined from the curve. By taking the ratio of the length of the wave to the length of the resonator as the ordinate, the ratio of the separation to the length of the resonator was the abscissa of the point on the curve. The distance between the ends of the resonators (E to E) does not affect the wave-length, hence this distance was made 10 or 15 mm. for all the screens. A number of screens were made up in this way and in all cases the measured and the calculated wave-lengths were in close agreement.

III. The Effect of a Change in the Width of the Resonators on the Wave-Length of the Energy Reradiated by Them.

It was now possible to take up the question of the effect of the width of the resonator on the wave-length. In this investigation it was found possible to determine the limiting dimensions of what may be called a "linear resonator." It was found that a change in the width changes the wave-length but slightly. The damping, however, and the amount of energy radiated by a set of resonators depend very largely upon the width of the resonators.

The effect of the width on the wave-length was first investigated by a method differing essentially from the multiple reflection method. The method was one of tuning, in which the primary source of the energy was tuned to the resonator screens. The energy which was transmitted, *i. e.*, passed through the screen, fell upon a receiver (non-selective): the tuning was judged from the relative amounts of energy absorbed when sources of different wave-length were used. This method was however unsatisfactory and had to be abandoned, as the effect on the wave-length of the changes in width is too small to be determined in this way. The method of multiple reflection was then employed.

Resonators 24 mm. wide and 36 mm. long were placed on screens with a distance between the ends of 10 mm. and a distance of 66 mm. between adjoining sides, *i. e.*, 90 mm. between centers of the resonators measured perpendicular to the electric oscillation. An interference curve was taken with the Boltzmann mirrors, with the energy successively reflected from two of these screens. The resonators were then made narrower by cutting equal amounts of foil from the sides of each resonator. The distance between the sides was in this way changed by the amount of foil taken from a resonator, the distance between the centers remaining the same. An interference curve was taken for the altered system of resonators. The resonators were then narrowed still more and the interference curve again taken. This process was continued until the resonators were too narrow to allow of further cutting from the sides. This set of screens will be referred to as "Set 1."

TABLE IV.

Width, Mm.	Wave-length Measured, Mm.	Wave-length Calculated, Mm.	Side-to-side Distance, Mm.
24	98	97	66
18	100	97	71
12	100	97	78
6	98.5	97	84
4	96.5	97	86
3	95	97	87
2	96	97	88

End-to-end distance.................................15 mm.
Center-to-center distance............................90 mm.
Length of resonators................................36 mm.
Wave-length of source...............................76 mm.
Two reflections used.

Table IV. gives the dimensions and distributions of the resonators for Set 1, and the observed wave-lengths taken from the interference curves. It will be seen from column I. that the width varied from 24 mm. to 2 mm., and from column II. that the wave-length varied from 95 mm. to 100 mm. only. The small variations in the wave-length do not follow the changes in the width. For example, with resonators having a width of 18 mm. the wave-length is 100 mm. while a width of 24 mm. gives a wave-length of 98 mm., a width of 3 mm. a wave-length of 95 mm. and a width of 2 mm. a wave-length of 96 mm. It is probable that this random apparent variation in the wave-length is due to errors of observation and does not represent a true effect. The greatest deviation of any value for the wave-length from the mean value is about 2.5 per cent., the average value of the wave-length being 98 mm. The greatest deviation between any two measured wave-lengths is only about 5 per cent. for a variation in the width of some 1,200 per cent. This small and random variation in the measured wave-lengths suggested to the authors that there was no very great effect on the wave-length caused by the change in width and further that the distance between the centers of the resonators, rather than the distance between the sides, was the important factor in determining the effect of separation perpendicular to the electric oscillation. This conclusion seems to be borne out by the results obtained from the screens of a second set, called Set 2.

The screens of Set 2 were made with resonators of the same width as those used in Set 1. With these screens the distance between the sides of the resonators was kept constant, necessitating that the distance between the centers of the resonators be varied. It was necessary with these screens to shift the resonators toward the center row each time the width was changed, by an amount equal to the change in the width of the resonators. The distance between the sides of the resonators was constant and equal to 54 mm. Table V. gives the results for the screens of Set 2.

TABLE V.

I.	II.		III.	IV.	V.
Width, Mm.	Wave-length		Center-to-center, Mm.	Wave-length of Source, Mm.	End-to-end, Mm.
	A Measured, Mm.	*B* Calculated,[1] Mm.			
0.3	84.6	83.9	54.3	76	10
2.0	84.6	84.6	56	76	10
3.0	83.5	85.0	57	76	10
5.0	85.0	85.0	59	76	10
9.0	86.5	87.5	63	76	10
18.0	91.0	91.0	72	76	10

[1] Calculated for "linear resonators" with same distribution as center-to-center distance of the wide ones.

Side-to-side distance constant and equal to 54 mm.

The assumption that the width does not greatly affect the wave-length and that the center-to-center distance is the important factor in determining the wave-length explains the results from the screens of Set 2 as well as those from Set 1. In the case of Set 1 this view requires that the wave-length be very nearly a constant, since the center-to-center distance was constant. And as was shown in Table IV. this was very nearly true, though the width was varied by several hundred per cent. Again this assumption requires for the screens of Set 2 that the wave-length correspond very nearly to the changes in the center-to-center distance between the resonators. This is also shown to be true, in Table V. (cf. column II., *A*, and III.). Other assumptions were made to explain these results and it was found that an assumption involving the width as an important factor and the distance between the sides as the important separation factor, necessitated the assumption of a relation between the wave-length, width and separation which was not so simple. On the other hand, the relation between the wave-length and separation on the assumption here made appears to be practically linear, a small correction being required only in case the width was large compared with the length of the resonator. From these results it appears that the width has but a small effect on the wave-length of the reradiated energy, and that the important factor to be considered is the distance between the centers of the resonators rather than the distance between the sides.

If the width does not greatly affect the wave-length, then the wave-length for a screen of non-linear (width comparable to the length) resonators should correspond rather closely to that for a screen of linear resonators for the same distribution. The separation distance for the linear resonators must be taken the same as the distance between the centers of the wide resonators. Calculating the wave-length which should be given by a screen of linear resonators arranged to have a *C* to *C* separation of 90 mm., the center-to-center distance for all screens of Set 1, it is found that the curve Fig. 2, *A*, requires a wave-length of 97 mm. for these linear resonators. The average value of the measured wave-length for these screens was 98 mm. In Table V. a column headed "calculated wave-length" (col. II., *B*) gives the wave-lengths for the linear resonators having the *C* to *C* separation corresponding to the center-to-center distance for the wide resonators. It is seen that these calculated values agree very well with the wave-length measured for the screen. Although the width was made to vary from 0.3 mm. to 18 mm. and the center-to-center distance from 54 mm. to 72 mm., the maximum value of the deviation of any one measured value from the corresponding calculated value is about 3 per cent. The actual variation in the wave-length is some 16 per cent.

All the results tend to show that the wave-length, or the natural period of the resonators, is very nearly independent of the width. That there is a slight variation due to the change in the width is shown by the fact that the curve for the effect of the separation in the case of the wide resonators is steeper than that for the linear resonators (see Fig. 2). But the variation of the wave-length for the square resonators from that for the linear ones having the same separation (*C* to *C*) is small. In the case when the separation is as great as can be tested by the multiple reflection method, the variation was only about 10 per cent. The change in width for this case was some 1,200 per cent.

Damping.—A change in the width of the resonators exerts a marked influence on the damping of the oscillation. This damping is evident from (1) changes in the amount of energy reradiated, (2) changes in the number of maxima and minima that may be taken, (3) changes in the purity of the radiation, or the "smoothness" of the interference curves. Table VI. shows the variation in the damping constant and amount of energy radiated as the width is changed. The damping constant[1] given

[1] Klemenčič u. Czermak, Ann. d. Phys., 1893, Vol. 50, p. 179.

in the last column was calculated as explained in the previous paper. This constant has a value of .15 for the narrow resonators and increases with the width up to a value of .31 for the square resonators. The damping constant of the radiation from the source has a value of 1.36.

TABLE VI.

Width, Mm.	Energy, One Reflection.	Energy, Two Reflections.	Damping Constant.
0.3	308 divisions	112 divisions	0.15
2	458	243	0.19
3	518	279	0.19
5	565	357	0.19
9	616	413	0.22
18	522	355	0.29
36	291	110	0.31
Strips	110	23	——
Source	——	——	1.36

With this highly damped source and total reflecting surfaces in the place of the resonator screens, it was possible to get only two or three maxima and minima in the interference curve. Using the narrow, linear resonators it was possible to take as many as fifteen or twenty such maxima and minima. That there is an effect on the purity of the radiation resulting from a change in the width is quite evident from a glance at interference curves for resonators of different widths and the same length. The curves become more and more irregular as the width is increased and secondary maxima and minima become more evident especially when the source is not well tuned to the resonators. With the wider resonators there seems to be a greater tendency of the source to force its own period on the resonators.

The energy increased with the width of the resonators until the width became about one fourth of the length, 9 mm. for the resonators used. If the width was increased beyond this value the energy decreased and when the width was equal to the length (square resonators) the energy was about the same as for the very narrow resonators. The increase in the width seemed to act as an increase in the number of resonators in the field. At the same time this increase in the number of resonators increased the damping of the oscillations in them. It appears that those factors which increased the damping also decreased the amount of energy reradiated. When the width has become as great as one fourth of the length the losses just balance the gain from the increased foil.

A rather extensive study was made of the resonators having their width equal to their length, *i. e.*, 36 mm. long and 36 mm. wide. It was found that if two reflections be used and the C to C separation be not greater than 2.5 times the length of the resonator, the interference curves were similar to those for the linear resonator; the damping however was greater. If the separation be greater than this the curves are very irregular and it is difficult to determine the wave-length from them. These irregularities in the curves become more pronounced as the separation is increased, and there seems to be a tendency for the resonator to oscillate with the period of the source. In order to assure good results and reliable interference curves, it is necessary that the incident energy have a wave-length of not less than 0.8 of the wave-length of the resonators, provided two reflections be used. With one reflection the interference curves are not very reliable, unless the source be very closely tuned to the period of the resonators. If the source have a wave-length of less than 0.7 of the wave-length of the resonators the radiation from one reflecting screen will probably correspond to that of the first overtone of the resonator.

It is to be expected that with resonators having as great a surface as the square ones there will be a certain amount of energy regularly reflected, as from a metal sheet. In order to obtain information on this point, a set of screens was made in such a way that this regularly reflected energy did not arrive at the receiving system. This was accomplished by rotating each resonator on the screens through an angle of about thirty degrees about a line through its axis parallel to the electric component of the oscillation. In this way the corresponding points in all the resonators were held in the same plane, so that the Huygens wave-front of the radiated energy was still a plane and the reradiated energy had the same path as before. The angle taken was more than enough to guarantee that this energy did not arrive at the receiving system. It was found that the interference curves were somewhat smoother, and that the energy was reduced by about 25 per cent.

The results from the square resonators therefore show that they are not very satisfactory as resonators. The damping is high, the energy is small and they show rather marked tendency to reflect the irregularities present in the incident energy. They also show a tendency to oscillate in their overtone when the tuning is not close.

In Table VII. is given the results of some work on resonators for which the width was greater than the length. The widths were varied from one to twenty times the length of the resonator. It will be seen that the wave-length measured for such resonators is in general very near to that for the source. It is evident that when the width is greater than the length there remains but little resonance effect, and there is no definite period for the resonators. The damping is high and the energy is small.

TABLE VII.

Width, Mm.	Length, Mm.	Wave-length of Source, Mm.	Center-to-center, Mm.	End-to-end, Mm.	Reflections.	Wave-length, Mm.
27	36	76	200	10	1	73
36	36	76	72	10	2	87
54	36	76	90	10	2	71
72	36	76	108	10	2	72
72	36	76	108	10	1	73
104	36	76	108	10	2	70
108	36	76	—	10	1	70
166	36	76	—	10	1	69
226	36	76	—	10	1	72.3
226	36	76	—	56	1	72
226	36	76	—	10	2	69
320	36	76	—	10	1	70
320	36	76	—	10	2	70
328	36	76	—	10	2	68.5
550	40	87	—	10	2	86
590	36	120	—	10	2	109
590	50	120	—	43	2	110
590	50	120	—	15	2	112
590	55	120	—	20	2	112
695	36	76	—	10	2	70.5

If a "linear resonator" means one whose wave-length does not change with a decrease in the width, then the results of this investigation show that a resonator may be considered "linear" so long as the width is less than one fourth of the length of the resonator. If the additional condition be imposed that the damping must also be independent of the width, the ratio of the width of the resonator to its length must be smaller than this value.

SUMMARY.

The object of the present investigation has been to determine the relation of the dimensions and distribution of the resonators to the resulting wave-length. The present study is an application of the "*Multiple Reflection Method*" described in the preceding paper by the same authors.

A summary of this investigation follows:

1. A change in the distance between the resonators measured along the direction of the electric oscillation has very little effect on the period of oscillation of the resonator. This applies to both linear and non-linear resonators.

2. For a screen of resonators the ratio of the wave-length of the energy radiated to the length of the resonators depends very largely upon the separation of the resonators measured perpendicularly to the electric

oscillation. The value of this ratio (K) is shown by the curves in Fig. 2 These curves may be used to determine the proper distribution of resonators to give any desired wave-length.

3. The effect on the wave-length of the distance between the resonators measured perpendicular to the electric oscillation is more simply expressed in terms of the center-to-center distance between the resonators than in terms of the side-to-side distance.

4. An increase in the width of the resonators of a given length has a very small effect on the wave-length reradiated.

5. An increase in the width of the resonators increases the damping of the oscillation. As a result the interference curves are irregular and the oscillation shows a more marked tendency than in the case of the linear resonators to correspond to that of the first overtone. For widths greater than one fourth of the length of the resonator, the energy decreases with an increase in the width.

6. If the width of the resonators be greater than 1.5 times the length, there is very little effect of resonance. The wave-length of the energy from the screen is determined almost entirely by the wave-length of the incident energy.

7. The ratio of the wave-length to the resonator length for a resonator not under the influence of other resonators has been shown to be greater than 2. The value estimated from this investigation is about 2.4.

8. It appears from the results of this investigation that any resonator having a width less than one fourth of its length may be taken as a "linear resonator." If the ratio be greater than this there is but a small change in the wave-length but a marked change in the character of the reradiated energy.

The authors take pleasure in expressing here their thanks to President E. F. Nichols, at whose suggestion these investigations were undertaken, and especially do they wish to express their appreciation for the many helpful criticisms and valuable suggestions of Dr. H. W. Webb throughout the work.

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Note.—Since the foregoing papers were sent to the publishers an article by K. F. Lindman has appeared in the Ann. d. Phys., April, 1913, p. 992. His results are, in general, in agreement with the results of the present work, although the methods are quite different.