THE EFFECT OF DIELECTRICS ON UNIPOLAR INDUCTION.

By E. H. Kennard.

In a recent article[1] Professor Barnett takes issue with the conclusions which I drew[2] from an experiment of my own on unipolar induction and which I should *a fortiori* draw from his results. In the Physikalische Zeitschrift I have already replied[3] to his chief criticism as presented in an earlier issue of that magazine,[4] but a further reply in connection with his later article may not be amiss.

Professor Barnett contends that neither my experiment nor his own decides the old question as to "motion of the force lines," but his argument appears to me to be fallacious in two particulars. The difficulty in both cases arises from the fact that on the moving force line theory e (the motional intensity) does not usually satisfy Laplace's equation.

In the first place, he assumes tacitly on p. 326 and explicitly on p. 327 that the electric density is proportional to div f. This was the assumption made by H. Hertz, who identified f (total intensity) and E (electric force) and rejected the motional intensity e. But on Lorentz's theory this assumption is not always allowable, and it may easily be shown to be incompatible with the moving line theory in the exact case mentioned by Professor Barnett on p. 327.

For suppose that the force lines there rotate with the magnet and let the surrounding dielectric be free ether. Then in the ether div E must vanish. But since $e = -[vB]$, where v = velocity of force lines,

$$\text{div}\, e = -(B\, \text{curl}\, v) + (v\, \text{curl}\, B),^5$$

or since

$$\text{curl}\, B = 0,$$

$$\text{div}\, e = -2(BU), \tag{1}$$

where U = vector angular velocity of lines. Thus div e and therefore div f = div $(E + e)$ will not usually vanish even where there is no electrification. I see no escape from this conclusion except by assigning to the ether some very peculiar properties invented ad hoc.

[1] Phys. Rev., Nov., 1912, p. 324.
[2] Phil. Mag., June, 1912, p. 937.
[3] P. ZS., Dec. 1, 1912, p. 1155; Mar. 15, 1913, p. 250.
[4] P. ZS., Sept. 1, 1912, p. 803.
[5] [ ] denotes the vector product, ( ) the scalar product.

In his latest article[1] on the subject Professor Barnett refers to discussions of unipolar induction by Poincaré and M. Abraham. Careful reading shows, however, that both of these authors approach the subject from the Hertzian standpoint and are thereby led virtually to make the same assumption as Professor Barnett concerning div. f. Furthermore, both authors dismiss the question as to whether the lines "move" or not with the observation that the lines are creatures of our imagination, so that the question has no sense. But neither author so much as mentions the substantial physical theory, apparently once a favorite in Germany, which employs the "moving lines" merely as a symbol, and which differs from Lorentz's theory chiefly in that in the equation

$$E' = E + 1/c[vB]$$

v is interpreted as velocity not relative to the ether but relative to the material system which is the source of the magnetic field. Hence for our purpose both of these references are really beside the point.

In the second place, in note (1) on p. 326 Professor Barnett does not state why $K - 1/K$ is to be replaced by unity. From the article there referred to, the reason seems to be that the moving lines are supposed to act on the ether (p. 433, top) in the same manner as on a material dielectric. But such an assumption is not a part of "current theory" as I know it, and I do not see how it is to be harmonized both with electrostatics and with the moving line theory. For simplicity, let us suppose that in the case cited above the magnet is electrically uncharged. Then if, as Professor Barnett assumes,

$$D = KE + Ke = Kf$$

we have everywhere div $D = 0$, curl $D =$ curl $e = 0$, and therefore D and f vanish at all points, together with the electricity density. Hence for electrostatic reasons div E should vanish, but it cannot do so because as shown above div e does not vanish. The only explanation in harmony with electrostatic theory would be the assumption that E itself contains a component due to the moving lines, but all such effects were included in e.

The real root of the difficulty appears to me to lie deeper yet. In the article to which the note refers Professor Barnett seems to treat the ether exactly like a material dielectric. But in electrostatics ether and matter are diametrically opposed: an outward displacement in matter (across a closed surface) leaves behind it in the matter a negative (free) charge, whereas an outward displacement in free ether leaves behind it a

[1] P. ZS., March 15, 1913, p. 251.

positive charge. And this difference is essential,—ether and matter are complementary: for if an outward displacement in matter is produced by any other cause than the insertion of a positive charge the ether must be free to take on a displacement backward corresponding to the change in E and serving to keep the total displacement unaltered.

Let us now combine with the moving line theory Lorentz's relation between D, E and e, and calculate the charge thus obtained on Professor Barnett's condenser. The result will likewise hold on this theory when the solenoid is at rest and the condenser rotates in the opposite direction, and in this case will also be true according to Lorentz's theory. We have in the short-circuiting wire $f = E + e = 0$, hence

$$E = -e = \omega Br,$$

where ω = angular velocity of lines (relative to condenser); and

(2) $$\psi = \omega B\frac{r''^2 - r'^2}{2},$$

where ψ = P.D. between inner and outer cylinders. In the dielectric

(3) $$D = KE + (K - 1)e,$$

$$\therefore (rD)\frac{dr}{r} = KEdr - (K - 1)\omega Brdr,$$

and since (rD) is constant we have after integrating and applying (2)

(4) $$D = \frac{\psi}{r \log \frac{r''}{r'}}.$$

Hence also

(5) $$E = \frac{1}{K}\left\{\frac{\psi}{r \log \frac{r''}{r'}} + (K - 1)\omega Br\right\},$$

(6) $$f = \frac{1}{K}\left\{\frac{\psi}{r \log \frac{r''}{r'}} - \omega Br\right\}.$$

The last equation agrees (except with $-\omega$ for ω) with Professor Barnett's calculation on p. 326 if the factor $K - 1/K$ is there retained.

Finally if σ_1, q_1 are surface and linear charges on inner cylinder, by (4)

(7) $$q_1 = \frac{2\pi\psi}{\log \frac{r''}{r'}}.$$

Since in the metal $f = 0$, $D = E = \omega Br$ and by (4)

$$\sigma_1 = \frac{\psi}{r' \log \frac{r''}{r'}} - \omega Br'.$$

If the inner cylinder is solid it contains a body charge $\rho_1 = \text{div } D$ or

$$\rho_1 = 2\omega B. \tag{9}$$

The charge on the condenser is thus independent of K, as is stated also by Professor Barnett. This result may be generalized as follows: If a rigid system composed of conductors and a homogeneous dielectric surrounded by a conducting surface, rotates about an axis of symmetry of a magnetic field (or is "cut by moving lines" which thus rotate), the resulting distribution of electrification is independent of the dielectric constant.

For under these conditions curl $f = 0$ and the line integral of f around any closed path, say $ABCDA$, where ABC lies in the dielectric and CDA in a conductor, is zero. But in the conductor $f = 0$, hence the integral of f along ABC must also be zero. Now $D = E + (K - 1)f$, hence the line integral of D along ABC equals the line integral of E, *i. e.*, equals the P.D., ψ, between A and C. But since along CDA $f = 0$, ψ equals the induced E.M.F. in CDA. Now also in the whole system curl $D = 0$ and in the dielectric div $D = 0$. Hence D is everywhere determined in terms of the induced E.M.F. in the conductors, and the electrification on the conductors is independent of the dielectric coefficient. That is, e in the dielectric merely cancels the polarization produced by the charges on the conductors.

This general theorem evidently applies to my own experiment, where a magnet rotated inside a closed conductor and an induced charge was sought (in vain) on a cylinder surrounding the latter. I think therefore that both Professor Barnett's results and my own do invalidite at least the simplest and most natural form of the old moving line theory.

Elsewhere I have called attention to the fact that Professor Barnett's own conclusion as to relative motion rests in part upon an inference. He deduces the existence of a charge in his Case I. from known laws combined with results of Blondlot, H. A. Wilson and Barnett on the effect of dielectrics moving in a magnetic field. This seems not quite conclusive because in the latter experiments sliding contacts (or a similar device) and stationary connecting wires that did not lie along the axis of the system were necessary adjuncts, and the dielectric was naturally not enclosed in a conducting screen that rotated with it. It seems possible that such a screen might prevent the production of any charges inside it. Such an effect is indeed required by the old theory of Hertz, which is however in other respects not in accord with the results of these experiments. It would thus seem desirable that Professor Barnett's experiment, fundamental as it is, should be repeated in such a manner that both positive and negative results could be obtained with the same apparatus.