A METHOD OF PRODUCING KNOWN RELATIVE SOUND INTENSITIES AND A TEST OF THE RAYLEIGH DISK.

By G. W. Stewart and Harold Stiles.

The absolute intensity of sound has been measured principally in four ways, viz., by the use of the Rayleigh disk placed directly in the sound,[1] by the measurement of the increased pressure at a reflecting wall,[2] by measuring pressure changes at nodes of stationary waves by a manometer,[3] and by optical interference methods.[4] In some of the experiments to which reference has just been made, varying measurable sound intensities have been produced, but in such a manner as to be unavailable for the calibration of intensity measuring devices of various kinds. Indeed, we have found no record of a successful effort to produce known varying intensities available for testing purposes. The application of the inverse square law is quite inaccurate, even out of doors. The construction of a sound-proof or a "silence" room will probably not reduce the reflection sufficiently to justify the assumption of the variation of the intensity inversely as the square of the distance.

The theory[5] of the acoustic shadow produced at any distance from a rigid sphere with the source located on the sphere suggested a method of producing known variations of intensity and thus obtaining a calibration device for sound-measuring apparatus.

The theory can be briefly stated. Let the source be confined to a small area on the surface of the sphere within which $P_n(\mu) = 1$. Let the velocity of this source region be simple harmonic and let it have the same magnitude U throughout. The following notation and equations are assumed:

ψ represents the velocity potential,
a represents the velocity of sound,
r represents the distance from center of sphere,
c represents the radius of the sphere,

[1] Zernov, Annal. d. Phys., 26, 1908, p. 79, Fig. 10.
[2] Altberg, Annal. d. Phys., 11, 1903, p. 405, and Zernov, Annal. d. Phys., 21, 1906, p. 131.
[3] Raps, Annal. d. Phys., 36, 1889, p. 273.
[4] Raps, Annal. d. Phys., 50, 1893, p. 193, and Sharpe, Science, 9, 1910, 1909, p. 808.
[5] Stewart, Phys. Rev., Vol. XXXVIII., No. 6, December, 1911.

dS represents an element of surface,

$$K = \frac{2\pi}{\text{wave length}},$$

$$\gamma = k(at - r + c),$$

$$F = \sum \frac{2n+1}{2} P_n(\mu) \frac{\alpha\alpha' + \beta\beta'}{\alpha^2 + \beta^2},$$

$$G = \sum \frac{2n+1}{2} P_n(\mu) \frac{\alpha\beta' - \alpha'\beta}{\alpha^2 + \beta^2},$$

$$^2f_n(ikr) = \alpha' + i\beta',$$

$$^2F_n(ikc) = \alpha + i\beta.$$

Then

$$\dot{\psi} = \frac{ka}{2\pi r}(F \sin \gamma + G \cos \gamma) \iint U dS. \qquad (1)$$

The energy per unit volume is $\frac{1}{2}\rho_0 a^2 s^2$, where ρ_0 is the density and s the condensation. But s equals $- \dot{\psi}/a^2$. Therefore we have for the mean potential energy the following expression,

$$\text{Energy per unit volume} = \tfrac{1}{2}\rho_0 \frac{\dot{\psi}^2}{a^2} = \tfrac{1}{2}\rho_0(F^2 + G^2)\left(\frac{k}{2\pi r}\iint U dS\right)^2. \qquad (2)$$

We determined to construct a sphere with a satisfactory source of sound thereupon, and thus to secure in the region of the sphere varying sound intensities of known relative values.

As shown in Fig. 1 the sphere was mounted on the edge of the roof of the Physics Building. It was placed on the side where the building was 21 meters high. There were neither buildings nor trees within several hundred feet, and this, combined with the high elevation, made the location very satisfactory. Indeed, the only reflecting surface was the roof. The arrangement of the apparatus shows that the error due to the reflection from the roof would be very small, and this error was further reduced by a covering of three fourths of an inch of hair felt.

The sphere was constructed of cement, the wall thickness being 5 cm. The diameter of the opening, the source of sound, was about 5 cm. The circumference of the sphere was 135.9 cm. As shown by the figure, the sphere could be rotated readily, the angle being indicated at the water seal. The sphere was supported by a horizontal 5 cm. pipe 230 cm. above it, and this pipe was in turn supported by two 5 cm. pipes and an iron flagstaff, all three being distant from the sphere at least 250 cm. The reflection from these supports was practically nil.

[1] $f_n(ikr)$ and $F_n(ikc)$ are defined in Rayleigh's Theory of Sound, Vol. II., and in article by Stewart, loc. cit.

The sound was produced by an electromagnetically operated tuning fork mounted on a resonator, the latter being introduced into a funnel located at a distance of 700 cm. from the sphere and connected to it through an iron pipe as suggested by Fig. 1. The frequency used was 256, and thus kc was very approximately unity.

The apparatus to the left of the sphere in Fig. 1 is a Rayleigh disk

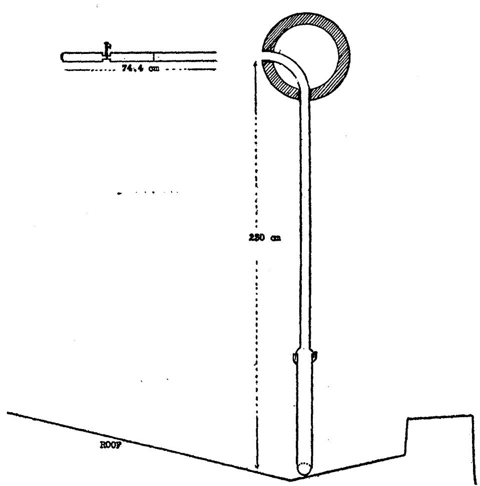

Fig. 1.

device. This was used to prove the practicability of this method of producing known relative intensities. This device is a modification of the one suggested by Rayleigh[1] and is drawn to scale in Fig. 2. It was made of brass tubing. The constriction in the tube was introduced to increase the sensitiveness. The dimensions needed were calculated by an approximate formula and then tested experimentally before constructing the apparatus.

The mirror, 0.6 cm. in diameter, was made from a very thin microscope

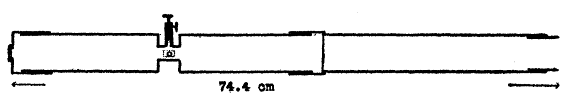

Fig. 2.

cover glass. It was suspended by a quartz fiber. The complete period was 6 seconds.

The observing telescope was placed along the axis of the tube and the scale parallel to the tube and in front of the mirror window. The apparatus had a high sensibility giving a definite deflection for what would be

[1] Rayleigh, Phil. Mag., Vol. XIV., p. 186, 1882.

termed a "faint" sound. The sensitiveness could have been increased greatly by reducing the size of the fiber suspension, but for work out-of-doors a short period was highly desirable.

The diaphragm was a piece of thin letter paper, but the selection of this material has no significance as it was not the result of investigation. The Rayleigh disk as used undoubtedly gives a deflection which is practically proportional to the kinetic energy on the interior at the disk itself. Indeed this has been demonstrated[1] experimentally. So far as we have been able to ascertain there is no experimental evidence that the kinetic energy at the disk itself is proportional to the potential energy which would exist at the opening of the resonating tube if the presence of the apparatus produced no distortion. Yet we here tentatively assume this to be the case. In this paper "energy" refers to potential energy unless otherwise designated.

We used the disk at three distances, viz., $kr = 2$, $kr = 3$, and $kr = 4$. The computations for these distances were made in accord with the formula (2) and the relative values of $F^2 + G^2$ obtained. The values for $f_n(ikr)$ and $F_n(ikc)$ were computed from equations defining these expressions.[2]

The values of the terms of Legendre's series $P_n(\mu)$ for the angles used from 0° to 180° were ascertained from tables and the relations

$$P_{2n+1}(90^\circ + \theta) = P_{2n+1}(90^\circ - \theta)$$

and

$$P_{2n}(90^\circ + \theta) = P_{2n}(90^\circ - \theta).$$

The terms were retained as far as $P_6(\mu)$. The computations are probably sufficiently accurate for the purposes of this paper. The results are shown in the accompanying tables.

The accompanying curves (Fig. 3 and Fig. 4) show these computations plotted with the value at 0° taken as unity. This is the position of the sphere when the source of sound is directly in front of the Rayleigh disk. The points indicated by small circles are the results of observations with the Rayleigh disk. The relative deflections are plotted. Our maximum deflection from the 45° position of the disk (*i. e.*, 45° between the normal to the mirror and the direction of the undisturbed stream) was 7°. An inspection of the formula[3] derived for the disk shows that the assumption of proportionality of energy to deflection does not introduce an error we need here consider.

[1] Zernov, Annal. d. Phys., No. 26, p. 79, 1908.

[2] Stewart, loc. cit.

[3] Konig, Wied. Annal., XLIII., 1891, p. 51.

TABLE I.

$kc = 1$, $kr = 2$.

	0°		30°		60°		90°	
	F	G	F	G	F	G	F	G
0	+0.2500	−.2500	+ .2500	−.2500	+.2500	−.2500	+.2500	−.2500
1	+ .7500	.0000	+ .6495	.0000	+.3750	.0000	.0000	.0000
2	+ .3020	+.2669	+ .1887	+.1669	−.0377	−.0334	−.1510	−.1335
3	+ .0949	+.1352	+ .0308	+.0439	−.0415	−.0591	.0000	.0000
4	+ .0398	+.0616	+ .0009	+.0014	−.0115	−.0178	+.0150	+.0231
5	+ .0188	+.0292	− .0042	−.0065	+.0017	+.0026	.0000	.0000
6	+ .0091	+.0142	− .0034	−.0053	+.0029	+.0046	−.0029	−.0044
	+1.4646	+.2571	+1.1123	−.0496	+.5389	−.3531	+.1111	−.3648
F^2+G^2	2.2111		1.2397		0.4151		0.1454	

	120°		150°		180°	
	F	G	F	G	F	G
0	+.2500	−.2500	+.2500	−.2500	+.2500	−.2500
1	−.3750	.0000	−.6495	.0000	−.7500	.0000
2	−.0377	−.0334	+.1887	+.1669	+.3020	+.2669
3	+.0415	+.0591	−.0308	−.0439	−.0949	−.1352
4	−.0115	−.0178	+.0009	+.0014	+.0398	+.0616
5	−.0017	−.0026	+.0042	+.0065	−.0188	−.0292
6	+.0029	+.0046	−.0034	−.0053	+.0091	+.0412
	−.1315	−.2401	−.2399	−.1244	−.2629	−.0717
F^2+G^2	0.0749		0.0730		0.0742	

The agreement between the theory of the acoustic shadow and the performance of the disk is not good in Fig. 3 for $kr = 2$, but is quite satisfactory in Fig. 4, $kr = 3$, conditions considered. If the disk itself were suspended in the open air it would give correct relative values of the mean kinetic energy per unit volume at the point. But the disk is enclosed in order to utilize the magnifying effect of resonance. This introduces several sources of error. A slight breeze interferes with the resonance of the tube, and doubtless this error has not the same relative value for all values of resonance. We worked under the best conditions obtainable and yet there was always a perceptible motion of the atmosphere. Our observations indicate that the small readings were greatly in error. It should be stated that the observed points are not averages of large numbers of readings, but represent different sets of observations. Another source of error is introduced by the absorption of energy by the resonating disk tube. This must disturb the distribution of sound intensity. It would seem that this distortion would tend to "iron out" the curve, or to produce higher readings on the steeper portions. One would also expect the distortion at $kr = 3$ to be less than at $kr = 2$.

TABLE II.

$kc = 1, \quad kr = 3.$

	0°		30°		60°		90°	
	F	*G*	*F*	*G*	*F*	*G*	*F*	*G*
0	+0.2500	−0.2500	+0.2500	−0.2500	+0.2500	−0.2500	+0.2500	−0.2500
1	+0.7000	+0.1000	+0.6062	+0.0866	+0.3500	+0.0500	0.0000	0.0000
2	+0.1311	+0.2904	+0.0819	+0.1815	−0.0164	−0.0363	−0.0656	−0.1452
3	−0.0122	+0.0876	−0.0039	+0.0285	+0.0053	−0.0383	0.0000	0.0000
4	−0.0082	+0.0212	−0.0002	+0.0005	+0.0024	−0.0061	−0.0031	+0.0079
5	−0.0026	+0.0057	+0.0006	−0.0013	−0.0002	+0.0005	0.0000	0.0000
6	−0.0008	+0.0017	+0.0003	−0.0007	−0.0003	+0.0006	+0.0003	−0.0005
	+1.0573	+0.2566	+0.9349	+0.0451	+0.5908	−0.2796	+0.1816	−0.3878
$F^2 + G^2$	1.1841		0.8760		0.4274		0.1834	

	120°		150°		180°	
	F	*G*	*F*	*G*	*F*	*G*
0	+0.2500	−0.2500	+0.2500	−0.2500	+0.2500	−0.2500
1	−0.3500	−0.0500	−0.6062	−0.0866	−0.7000	−0.1000
2	−0.0164	−0.0363	+0.0819	+0.1815	+0.1311	+0.2904
3	−0.0053	+0.0383	+0.0039	−0.0284	+0.0122	−0.0876
4	+0.0024	−0.0061	−0.0002	+0.0005	−0.0082	+0.0212
5	+0.0002	−0.0005	−0.0006	+0.0013	+0.0026	−0.0057
6	−0.0003	+0.0006	+0.0003	−0.0007	−0.0008	+0.0017
	−0.1184	−0.3040	−0.2709	−0.1824	−0.3131	−0.1301
$F^2 + G^2$	0.1065		0.1065		0.1150	

This expectation seems to be realized in the observations as shown in Figs. 3 and 4.

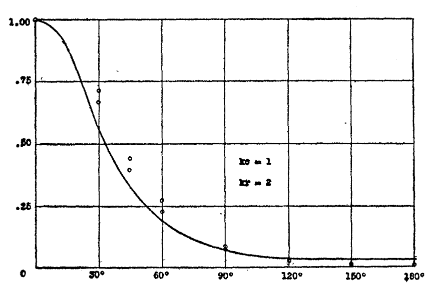

Fig. 3.

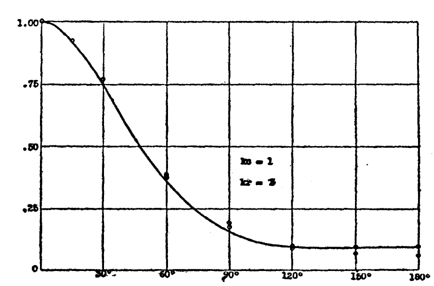

Fig. 4.

The theory, equation (2), shows that the intensities at different distances are proportional to $(F^2 + G^2)/r^2$. The results of a brief test at three different distances, $kr = 2$, $kr = 3$ and $kr = 4$, are presented in the accompanying table.

Table III.

kr	$F^2 + G^2$	$A = \frac{F^2 + G^2}{r^2}$.	B = Deflection.	Ratio of A to B.
2	2.206	0.551	11.80	21.4
3	1.145	0.127	2.80	22.0
4	0.915	0.0572	1.28	22.4

The theory is verified in that the ratio between the observed and theoretical relative values is practically constant. It is interesting to note that if the observed values are tested in a similar manner, but assuming the inverse square law, the results for the last column differ as much as 130 per cent.

The results presented in this paper certainly demonstrate that the method of producing known relative sound intensities is a practicable one, although attended with some difficulty of operation and limited both by the absorption of the instrument to be calibrated and the inconstancy of the source of sound. So far as the experiments with the Rayleigh disk are concerned, the results may be regarded in either of two ways. One may consider that they show the theory to be correct, assuming the deflection of the Rayleigh disk to be proportional to the energy. The writers, however, regard the theory of the acoustic shadow as more reliable than the assumption as to the action of the disk. The experiments call attention to certain errors in the operation of the disk which are unavoidable in any measuring instrument which utilizes resonance.

Physical Laboratory,
State University of Iowa.