AN ABSOLUTE DETERMINATION OF THE VISCOSITY OF AIR.

BY LACHLAN GILCHRIST.

IN view of the recent work of Professor R. A. Millikan, of the University of Chicago,[1] the accuracy with which the elementary charge can be determined is limited only by the accuracy obtainable in the measurement of the absolute value of the coefficient of the viscosity of air. It is of the utmost importance therefore for this reason, and also in view of the recent work that has been carried out on the relation between temperature, pressure and viscosity, that the absolute value of this constant be determined with all possible precision. Professor Millikan had already been led by a careful study of existing data to the conclusion that the present uncertainty in η for air at 25° C. could be still further reduced by new work with a constant deflection method since this method is exceedingly direct and simple and seems more suitable for the absolute determination of η than are either the capillary tube or oscillating body methods. At his suggestion therefore and with his assistance the following apparatus was designed. The results show that the method is capable of measuring η with an uncertainty which does not exceed .2 per cent.

The constant deflection method does not appear to have been used carefully in a direct determination of η for air except by Zemplen.[2] He used concentric spheres separated by a small distance and his results are much higher than those which have been obtained in any trustworthy experiments in this field.

In the present work it was decided to use concentric cylinders for two reasons.

1. It appeared to be the simplest method and the one in which the sources of experimental error could be most readily reached and their effect minimized.

2. The theory associated with the method is simple and the formula which is derived is exact.

[1] PHYS. REV., Vol. XXIII., No. 4, April, 1911.

[2] Annalen d. Phys., 29, 5, pp. 869–908, Aug. 10, 1909; 38, 1, pp. 71–125, May, 1912.

A. Theory.

The theory shows that

$$\eta = \frac{\pi \varphi I(b^2 - a^2)}{a^2 b^2 T^2 \omega l},$$

where η = the coefficient of viscosity.
I = the moment of inertia of the suspended body about the line of suspension.
T = the period of oscillation of the suspended cylinder.
φ = the angular displacement of the suspended cylinder.
a = the radius of the inner cylinder.
l = the length of the inner cylinder.
b = the radius of the outer cylinder.
ω = the constant angular velocity of the rotating cylinder.

The inner cylinder itself was used to determine I and T, and φ was determined by a mirror and scale and a telescope for observation.

Then

$$\eta = \frac{stK}{dT^2},$$

where d = the distance of the scale from the mirror.
$s = 2\varphi d$, that is, the observed deflection of the scale image.
$t = 2\pi/\omega$, that is, the time of one rotation of the outer cylinder.

Now it was hoped from the use of a bifilar suspension to diminish such errors as are due to fatigue or stretching of the suspension.

With such form of suspension the expression for the period is

$$T = \frac{2\pi k}{\alpha}\sqrt{\frac{l}{g}},$$

for small oscillations, where k is the radius of gyration of the cylinder.

α = one half the distance between the strands of the suspension.
l = the length of the suspension.
g = the gravity constant.

A consideration of these formulæ suggested the form of construction and the dimensions used in the following apparatus:

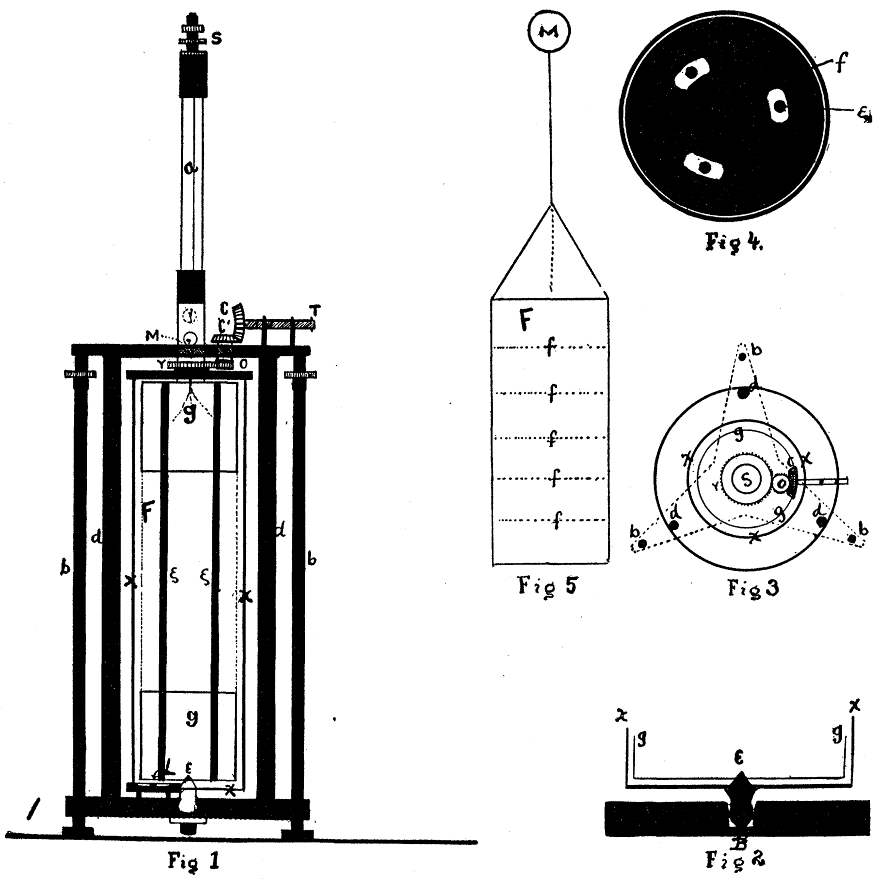
B. Experimental Arrangements.

Figure 1. Main Apparatus.

F, the inner cylinder with suspension and mirror M attached.
The length of the inner cylinder = 24.88 cm.
The outside radius of the inner cylinder = 5.342 cm.
The length of the suspension = about 25 cm.

This cylinder was made of brass, the wall being made as thin as possible to diminish the mass. The wall was supported by sheets of aluminium as shown in Figs. 5 and 4, *ff*.

The wall of the cylinder was made parallel to the line of suspension in the following manner: The center of the upper aluminium sheet was



Figs. 1–5.

carefully determined to within .1 mm. A fine steel wire suspension was temporarily attached at the center of the upper sheet and the cylinder weighted until the wall at all points was parallel. This was tested with a plumb square to which a very sensitive level was attached. Then three aluminium supports for the permanent suspension were fastened to the upper aluminium sheet and just above their junction the point of attachment of the suspension was placed. This point could be readily adjusted to make the line of suspension parallel to the wall of the cylinder.

The mass of the inner cylinder was 320 grams.

S = the screw head to which the suspension was attached.

gg were guard cylinders about 10 cm. long and rigidly attached by three brass rods. These were also rigidly attached to the tube supporting the screw head S. The distance between the guard cylinders was 24.92 cm., leaving .4 mm. between the suspended cylinder and the guard cylinder. The lower guard cylinder engaged the outer cylinder at ϵ as is shown.

a is a glass portion of the tube so that the suspension thread was always in view.

XX = the outer cylinder, *i. e.*, the rotating cylinder. The inner radius = 6.06357 cm.

This cylinder engaged the frame at Y and ϵ as is shown.

The lower bearing is shown in fuller detail in Fig. 2.

$CC'OY$ represent the transmission gear connecting the driving shaft T with the outer cylinder.

L = a level. There were two at right angles to each other.

d = the supports of the frame

b = the levelling rods on which the instrument was supported.

Fig. 2 shows the lower bearing of inner guard cylinder g, the outer cylinder X and the frame B.

Fig. 3. A plan of the top of instrument.

S = the opening through which the tube which contained the suspension passed.

$CC'Oy$ the transmission gear.

g = the inner guard cylinder, as given in Fig. 1.

x = the outer cylinder as given in Fig. 1.

b, d = etc., as given in Fig. 1.

The dotted lines indicate the supporting frame at the top of the instrument.

Method of Measuring Dimensions.

1. *Inner Cylinder.*—The length and diameter were obtained by means of calipers reading to .001 cm. and the average of a large number of readings was taken.

The mass was obtained by means of a balance reading, .1 mg.

2. *Outer Cylinder.*—The diameter was obtained by filling the cylinder with water, weighing the water, noting the temperature and using density tables to get volume, and then calculating the diameter.

3. The time of rotation of the outer cylinder and the period of oscillation of the inner cylinder were obtained by means of a stop watch. This watch was compared with a chronometer which had been regulated to

accord very closely to the clock in the Canadian Government's Observatory at Toronto.

4. The thermometer which was graduated to .1 dg. was compared with the standard thermometer in use at the observatory. A Beckmann thermometer was also immersed in the bath to indicate changes of temperature.

Fig. 2. Other Apparatus.

1. A clock-driving apparatus was used to drive the rotating cylinder. This instrument was of superior type and simple construction. It was driven by weights and could be directly connected by a shaft to the cylinder. It was made by Wm. Gaertner & Co., of Chicago, and was kindly loaned for the experiment. The average error over one hour in the clock did not exceed 1 part in 3,600. When it was loaded and set running the tests for speed agreed to this degree of accuracy.

The period of vibration of the inner cylinder was such that an average error in the rotation of the outer cylinder much larger than this would not seriously affect the steadiness of deflection.

2. A constant temperature bath. This was a water-bath of the kind described in several text-books in physics.

It was heated by incandescent electric lamps and was regulated by a mercury thermostat, a telegraph relay being used to control the make and break of the heating circuit. For temperatures within 10° C. of the temperature of the room when making a determination of η, the bath did not vary .01° C. and not more than .1° C. at 20° C. above the temperature of the room.

3. *The Telescope and Scale.*—The scale was ruled carefully for the experiment by Wm. Gaertner and Co. and the telescope was such that readings to .1 mm. could be made fairly readily.

The complete apparatus was set up in the first place on a stone pier in Ryerson Physical Laboratory, University of Chicago, during the summer of 1910 and subsequently on a stone pier in the basement of the Physics Building, University of Toronto. In this way rigidity and constant temperature were obtained and drafts and other disturbances were eliminated.

C. The Method of Measurement and Observation.

1. The following was the general plan of work including the difficulties which were encountered, and the methods of dealing with them.

The inner suspended cylinder was made plumb with the line of suspension by use of a plumb square. A shift of approximately five divisions in the level corresponded to .05 mm. out of plumb between top and bottom. One tenth of one division on the levels could be observed.

The levelling rods on the instrument were adjusted so that the outside surface of the guard cylinders, *g*, Fig. 1, was aligned with the surface of the inner cylinder. Two short focal length telescopes which were adjustable on the same base were used to observe this condition. The levels on the instrument were then adjusted. There were scales as well as cross hairs in the telescopes. One half of one division on the scale was easily recognized and the distance between the divisions represented .05 mm. motion of the telescope. The cylinder was found to be from .05 to .15 mm., less in diameter than the guard rings, being smaller at lower end than at upper.

The suspended cylinder was set very accurately symmetrical to the guard cylinders in this way. The upper end of the suspended cylinder was made symmetrical with the upper guard cylinder by adjusting the horizontal screws at the upper end of the suspension. Then the lower end was made symmetrical with the lower guard cylinder by the use of the levelling screws. This threw the upper end out a little. It was readjusted, and again the lower end, and so on. It will be observed that this plan really adjusted the instrument to the suspended cylinder which had previously been made plumb. The levels were then made to read properly. Then the suspended cylinder was lowered to rest on the lower guard cylinder one quarter to one half turn of the screwhead being sufficient for this purpose. The mounted telescopes were also used to test the symmetry of the guard cylinder about the axis of rotation. The average variation was about .05 mm.

2. The accuracy of centering of the outer cylinder was tested thus. The surfaces of the guard cylinders and the inner cylinder were put in alignment as in (3) above and the levels *L* adjusted. The suspension tube *a* and the inner cylinder *F* were then removed. An upright rod with arms at each end which were attached perpendicularly to it was inserted through the opening that was left by the removal of the suspension tube. This rod was arranged so that it could be raised or lowered or rotated in a horizontal plane, and by means of a swivel any motion of the lower arm was communicated to the upper arm. The extent of the motion was indicated by a sensitive level attached to the upper arm. The end of the lower arm was held lightly in contact with the inside wall of the outer cylinder by a small weight attached to the upper arm and the cylinder was rotated by the clock driving apparatus. The test for different places in the cylinder showed an average variation of less than .05 mm., the range of variation being very small. The instrument was adjusted and fixed solidly in the bath and the following observations were made, viz., the zero position on the scale, the period of oscillation of the

inner cylinder, its deflected position, the time of rotation of the outer cylinder, and the distance from the scale.

Some difficulty was found with the suspension owing to its elastic properties and the effect of temperature and moisture—silk, rolled and unrolled phosphor bronze wire, and rolled and unrolled steel wire of different sizes were tried. The use of silk was not successful. The others were used both as single strand and as bifilar suspensions and fair results were obtainable with several of the suspensions used, but the phosphor bronze ribbon rolled from wire, .0762 mm. in diameter and used as a bifilar suspension, gave the best results. It was necessary to obtain a suitable deflection for accurate measurement and still maintain constant zero and deflected positions of the cylinder. With several of the suspensions there was a slow drifting of the zero position or a slight unsteadiness of the deflected position. It was found that upon making the suspension bifilar and such that the period of oscillation of the cylinder was long, a very steady deflected position was obtainable. By leaving the instrument in the constant temperature bath for several hours with the inner cylinder suspended the zero position became more steady. It was also arranged that the driving apparatus could be damped so that it commenced slowly and gradually reached the condition of uniform speed. In this way the deflected position was quickly reached and oscillations that otherwise would exist, were eliminated. The measurement of the deflection was then made and the cylinder was gradually let back to the zero position. With this proceedure deflections of more than 600 mm. could be repeatedly duplicated with an error of no more than .3 mm.

In each determination all the observations and measurements were checked by repetition.

II. *The Measurement of the Moment of Inertia of the Inner Cylinder about the Line of Suspension.*

(*a*) The cylinder was suspended by a strand of steel piano wire .36 mm. in diameter from a firmly built support resting on a stone pier. It was set up in a room which was free from drafts and whose temperature was maintained constant to within 5° C. The cylinder was set oscillating for some time. The period was then obtained by counting a large number of oscillations of small amplitudes, noting the full time, and then calculating the time of a complete oscillation.

(*b*) An annular ring with a cross bar of aluminium was made in the workshop of Ryerson Physical Laboratory with dimensions uniform to a high degree of precision. This was placed symmetrically on the cylinder and the period of oscillation was again obtained, the amplitude of the oscillations being almost the same as those considered in (*a*).

The expression for the period in (*a*) is

$$T_1 = 2\pi\sqrt{\frac{I_1}{G}}, \tag{1}$$

and in (*b*)

$$T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{G}}, \tag{2}$$

where T_1 = period of oscillation of the cylinder alone.
T_2 = period of oscillation of the cylinder and the aluminium ring and bar.
I_1 = moment of inertia of the cylinder.
I_2 = moment of inertia of the ring and bar. This was calculated from the dimensions.
G = the couple constant of the suspension.

From (1) and (2) I_1 was obtained.

(*c*) *Calculation of* I_2.—For the aluminium bar

$$I_2' = M\frac{c^2 + d^2}{3},$$

where M = the mass of the bar = 22.157 grams.
c = the half width of bar = 6705 cm.
d = the half length of bar = 9.2265 cm.

For the aluminium ring

$$I_2'' = M'\frac{e^2 + f^2}{2},$$

where M' = the mass of the ring = 97.3771 grams.
e = the radius of outside edge = 9.2075 cm.
f = the radius of inside edge = 8.1608 cm.

$$I_2 = I_2' + I_2'' = 8002.409.$$

The periods of free oscillation of the cylinder without and with the ring and bar were 19.7183 secs. and 28.239 secs. respectively.

$$\therefore I_1 = 7615.8.$$

Then

$$K = \frac{I(a^2 - b^2)}{4a^2b^2}$$

$$= 7615\cdot 8\frac{[(5\cdot342)^2 - (6.06357)^2]}{4 \times (5\cdot342)^2 \times (6.16357)^2 \times 24.88} = .60027.$$

D. Results.

I. *Measurements of* η.

The coefficient of viscosity was then calculated from the formula

$$\eta = \frac{stK}{dT^2}.$$

In Table I. are given the values obtained with different speeds of rotation. It shows that the experimental method that has been used gives results which are very concordant. Save for the last two observations which deserve less weight than the others owing to the smallness of the deflection, the maximum divergence of any observation from the mean is but .22 per cent. and the average divergence but .1 per cent.

Table I.

	t	s	d	T	$st/dT_2 \times 10^7$	$\eta \times 10^7$	Temp.	$\eta \times 10^7$ at 20.2° C. (Calc.).
1	30.89	70	316	150.33	3,027	1,817	20.2	1,817
2	30.88	69.86	316	150.32	3,021	1,813	20.205	1,813
3	30.79	59.92	302.55	142.06	3,021	1,813	20.2	1,813
4	61.74	34.87	316	150.32	3,015	1,810	20.2	1,810
5	61.66	31.74	315.35	147.46	3,015	1,810	20	1,811
6	61.60	30.01	302.55	142.11	3,025	1,815	20.2	1,815
7	61.58	30.04	302.55	142.11	3,027	1,817	20.2	1,817
8	123.44	16.72	315.35	147.26	3,018	1,812	20.1	1,812.5
9	123.22	16.75	315.35	147.32	3,015	1,810	20.1	1,810.5
10	184.96	11.09	315.35	147.31	2,997	1,800	20.1	1,800.5
11	184.88	13.86	301	188.25	3,007	1,805	20.1	1,805.5

These results for η have been reduced (see Table IV., last column) to the value for the common temperature 20.2° C. by the use of the formula used by Professor Millikan[1] for small differences in temperature.

$$\eta_t = .00017856\,[1 + .00276(t - 15)].$$

The most probable value of η at 20.2° C. is obtained as follows: Since the deflections for the higher speeds of rotation are proportionally greater than for the lower speeds the values should be weighted accordingly. Considering also the steadiness of the deflected and undeflected positions in each case it was decided to give the readings the following weights: 1, 2, 3, 4 each 2; 5, 6, 7 each 1.5; 8, 9, 10, 11 each 1.

This gives as a mean value $\eta = 1{,}812\ (2) \times 10^{-7}$, with a probable error less than 1.3×10^{-7}. The total probable percentage error may also be fairly closely estimated. The probable error of observation as here

[1] Reference already given.

calculated is about .14 per cent. Since however for the different measurements which are given the apparatus was in most cases replaced and readjusted this would include the errors of observation in the measurement of s, t, d, T, and the effect on these of the error of adjustment. The errors in the measurement of a, b and l are exceedingly small and together with that in the measurement of the moment of inertia, do not, I believe, exceed .06 per cent. Then the total probable error is not greater than .2 per cent.

II. In order to determine whether this method would give the same temperature coefficient of viscosity as that which has been obtained by the capillary tube method the following observations were made (see Table II.). For the purpose of comparison the values which are obtained by calculation from the formula of S. W. Holman[1] $\eta \times 10^7 = 1715.50 (1 + .00275t - .00000034t^2)$ are also given. This formula has been shown by several experimenters to be very satisfactory and was found by Tomlinson[1] to accord with his results perfectly.

TABLE II.

Temp.	t	s	d	T	$st/dT \times 10^7$	$N \times 10^7$	$N \times 10^7$, Using Holman's Formula (Calc.).
20.2						1,812.2	1,810
24.58	30.86	66.58	316.5	146.28	3,034.5	1,821.5	1,831
31.9	30.86	67.8	316.5	146.25	3,090.7	1,855.2	1,865.4
37.9	30.88	68.15	316.5	145.84	3,126.2	1,876.5	1,892
43.5	30.88	69.49	316.5	145.84	3,126.2	1,913.4	1,919.6

The greatest variation of the observed from the calculated values is .82 per cent. and the curve showing the relationship between η and temperature is not quite so steep as that given by Holman's formula, or by the formula obtained by most of the experimenters with capillary tubes. The range of temperature for my observations is not great enough, however, to give the relationship as closely as is desirable nor are the determinations at the higher temperatures as accurate as that at 20.2° C.

III. *The Effect of Moisture on the Viscosity of Air.*

The following observations were made in order to determine whether the formula for mixtures of gases which is based on the kinetic theory holds for a mxture of water vapor and air.

[1] Reference given in Table IV.

Air which was dried by being passed slowly through phosphorous pentoxide and sulphuric acid was passed for some time into the instrument through a tube reaching to the bottom. Then a determination of the viscosity was made. Some of the air from the instrument was then drawn slowly into a Crova[1] dew point apparatus and the dew point observed.

The results obtained are shown in Table III. For the last three determinations water was passed into the outer cylinder for the purpose of saturating the air. In number 5 only 2 to 3 c.c. were put in and in numbers 6 and 7 water was put in to a depth of about 2 cm.

The Crova dew point apparatus could not be used to show that the air in the instrument was saturated since the temperature of the room was about 18.3° C. and that of the air in the instrument about 20° C.

TABLE III.

	t	s	d	T	Temp.	Dew Pt. Point.	$st_2/dT \times 10^7$.	$\eta \times 10^7$.
1	3,086	6,557	316.5	145.66	20.02°C.	6.6	3,015	1,810
2	3,088	6,552	316.5	145.34	20.05	11.6	3,026	1,816
3	3,088	6,557	316.5	145.57	20.05	14.1	3,017	1,811
4	3,087	6,546	316.5	145.62	20.02	17	3,010	1,807
5	3,081	6,432	316.5	143.84	20.02		3,026	1,816
6	3,083	6,432	316.5	144	20		3,021	1,813
7	3,086	6,438	316.5	143.82	20		3,034	1,821

Temperature of room = 18.2° C.

From the results in Table III. it does not appear that the presence of unsaturated water vapor in the air has any measurable effect on the coefficient of viscosity at normal pressure. It is noteworthy, however, that the average of the first four results gives $\eta = 1{,}811 \times 10^{-7}$ and the average of the last three, for which the maximum quantity of water was present, and saturation apparently existed, gives $\eta = 1{,}816.5 \times 10^{-7}$. This seems to show an increase of about .3 per cent. This result is smaller than that obtained by Zemplen[2] (viz., .8 per cent.) but agrees more closely with that obtained by Tomlinson. From the Puluj[2a]

[1] See Preston, Theory of Heat.

[2] Reference already given in paper. (See 2 above.)

[2a] $$\eta = \eta_1 \frac{\sqrt{p_1 + \frac{m_2}{m_1} p_2}}{\left[p_1 + \left(\frac{\eta_1}{\eta_2}\sqrt{\frac{m_2}{m_1}}\right)^{\frac{2}{3}} p_2\right]^{\frac{3}{2}}},$$

where η = coefficient of viscosity of the mixture, $\eta_1\eta_2$ = coefficient of viscosity of the components, p_1p_2 = partial pressures of the components, $p_1 + p_2 = 1$, m_1m_2 = molecular weights of the components.

formula for mixtures of gases there should be a decrease of about 2 per cent. A modification of this formula by Thiesen[1] has recently been shown by Tanzler[2] and Thomsen[3] to hold fairly well for certain gaseous mixtures. For saturated water vapor the formula is not sustained by the results of my experiments. These accord much better, however, with the calculations of Tomlinson by the method of Stokes[4] based on the experiments of Crookes according to which there should be an increase in the coefficient of viscosity of about .2 per cent. Crookes[5] found that the presence of water vapor did not affect the coefficient of viscosity of air until the total pressure was less than half of the normal pressure and for pressures lower than this it was diminished but not as much as is required by the kinetic theory on which the Puluj formula for gases is based. The results of recent experiments on mixtures of gases by Tanzler and by Kleint[6] are as much as 1.3 per cent. higher than the values calculated from the Puluj formula, that is, the variation is in the same direction as that which appears to exist for mixtures of water vapor and air. It is very desirable then to have the work on mixtures of gases carried out with the same proportional partial pressures as have been investigated, but through a range of lower total pressures until at least the critical pressure is reached. Thomsen shows that in the case of the mixtures of gases investigated by Tanzler and by Kleint, the results which were obtained were approximately such as were to be expected from the formula of Puluj, and Schmitt[7] succeeds in making the approximation much closer by means of a correction factor made necessary by the method of experiment which was used, the formula for which had been developed by Knudsen[8] from kinetic theory considerations. This however makes more imperative the necessity for the investigation which has been suggested above.

E. The Work of Other Experimenters.

The results of recent work by the oscillating body and capillary tube methods are given in Table IV. Those of Tomlinson by the former method and of Holman by the latter method, though of more remote date, are also given, as their work shows every evidence of great care and a high degree of precision. The values at 20.2° C. are calculated from the

[1] M. Thiesen, Ver. der Deut. Phys. Ges., 8, 12, 1906, p. 237.
[2] P. Tanzler, Ver. der Deut. Phys. Ges., 8, 12, 1906, pp. 222–235.
[3] E. Thomsen, Ann. d. Physik, 36, 1911, p. 815.
[4] G. F. Stokes, Phil. Trans., Part II., 1889, p. 440.
[5] Crookes, Phil. Trans., Part II., 1889, p. 427.
[6] F. Kleint, Ver. der Deut. Phys. Ges., 7, 1905, pp. 145–157.
[7] K. Schmitt, Ann. d. Physik, 4, 30, 1909, pp. 393–410.
[8] M. Knudsen, Ann. d. Physik, 4, 28, 1909, pp. 75–130.

TABLE IV.

Experimenter.	Method.	Temp.	Result $\times 10^7$.	Value at 20.2° C. $\times 10^7$.	References.
1. S. W. Holman	Capillary tube			1,810	Phil. Mag. (5), 1886, p. 199. $\eta = .00017155. (1 + .002751t - .00000034t^2)$
2. W. J. Fischer	Capillary tube			1,807	PHYS. REV., 28, p. 104.
3. J. H. Grindlay and A. H. Gibson	Capillary tube	18.94	1,802.7	1,809	Proc. Roy. Soc., A, 80, 1908.
4. A. O. Rankine	Capillary tube	10.6	1,767	1,814	Proc. Roy. Soc., A, Vol. 83, 1910, p. 522.
5. Rapp	Capillary tube	26.	1,838	1,810	Not yet published.
6. Breitenbach	Capillary tube	15.	1,817	1,833	Wied. Ann., LXVII., 1899, p. 803.
7. Schultze	Capillary tube	15.	1,811	1,837	Ann. d. Phys., Vol. 5, 1801, pp. 166–169.
8. Markowski	Capillary tube	15.9	1,814	1,835	Ann. d. Phys., Vol. 14, 1904, p. 742.
9. Tanzler	Capillary tube	15.	1,812	1,838	Ver. der Deut. Physik. Gesell., 8, 12, 1906, pp. 222–235.
10. H. Tomlinson	(*a*) Oscillating pair of cylinders.	12.65	1,774	1,811	Phil. Frans., Vol. 177, Part II., 1886, pp. 767–789.
	(*b*) Oscillating pair of spheres.	9.97	1,762	1,812	
	(*c*) A single oscillating cylinder.	11.78	17,711	1,812	
11. J. L. Hogg	....	17.33	1,794	1,808	Am. Acad. Proc., 40, 18, 1905, pp. 611–626.

values actually obtained. For all except that of Holman and Fisher the mean values for temperatures near 20.2° C. were taken and the transformation was made by means of the formula used by Millikan.

The Holman and Fisher values are calculated from their final formulæ.

The results are grouped as follows:

(*a*) Those which are obtained by the capillary tube method in which the tubes that were used were fairly large and the results obtained were concordant and agree well with the result obtained by me. (See nos. 1–5.)

(*b*) Those which were obtained by the capillary tube method in which the results, though fairly concordant differ considerably from those of the others. (See nos. 6–9.)

(*c*) Those which were obtained by the damping of a simple pendulum in which all or nearly all of the loss of energy is due to the pushing of the air. (See nos. 10, *a* and *b*.)

(*d*) The damping of torsional vibrations where the complete loss of energy is due to *dragging*. (See nos. 10, *c*, and 11.)

It will be seen from the table that in the work with capillary tubes the results obtained by Breitenbach, Schultze, Markowski, Tanzler and Schmitt are somewhat higher than those of Holman, Fisher, Grindley and Gibson, Rankine, and Rapp, through the determinations by both groups seems to have been made with equal care. With the oscillating body method exceedingly careful work has been done by Tomlinson and recently by Hogg. Their results are concordant but differ considerably from that of Reynolds[1] who used a similar method. In view of the care with which the work has been done it would appear that the discrepancies can only be accounted for by the fact that there are factors in these methods, the effects of which are somewhat indefinitely determinable and therefore have not been uniformly estimated.

It may be well to point out some of the difficulties and sources of error associated with each method and which may to some extent account for the difference in results.

1. *Variations in the Form of the Bore.*—The following will illustrate. The equation giving the coefficient of viscosity for a tube of circular bore as developed by O. E. Meyer[2] is

$$\eta = \frac{\pi g d r^4}{16LV} \frac{P^2 - p^2}{P} t, \qquad (1)$$

where d = density of mercury at 0° C.

[1] F. G. Reynolds, Phys. Rev., 19, 1904.

[2] O. E. Meyer, Kinetische Theorie der Gase, 1899.

g = acceleration due to gravity.
r = radius of the capillary in cm.
L = length of the capillary in cm.
P = pressure on entering the capillary.
p = height of barometer, *i. e.*, the pressure at 0° C. at exit.
y = time in secs.
V = volume of gas transpired.

This is the expression usually made use of. Now in case of a tube of elliptical bore the formula developed by Émile Mathieu[1] is

$$\eta = \frac{\pi dg}{8LV}\frac{a^3b^3}{a^2+b^2}\frac{P^2-p^2}{P}t, \tag{2}$$

where a and b are the semi-major and semi-minor axes respectively. Now for two tubes one with circular and the other with elliptical bore through which the same volume of gas was transpired and which had the same rate of shear, the areas of cross section would be the same; *i. e.*,

$$r^2 = ab. \tag{3}$$

No. (1) is greater than (2) if

$$\frac{r^4}{2} > \frac{a^3b^3}{a^2b^2},$$

or by (3) if

$$\frac{a^2+b^2}{2} > \frac{a^3b^3}{a^2b^2},$$

then

$$a^2 + b^2 > 2ab,$$

$$(a-b)^2 > 0,$$

and therefore the square of the difference between the semi-major and the semi-minor axes is a measure of the error introduced.

If a tube with an elliptical bore is used and the calculation is made for a tube with a spherical bore of the same cross section the result will be too high. This is illustrated in the results of experiments with a tube of circular bore and a tube of elliptical bore carried out by F. M. Pedersen.[2]

2. The difference in the form of the lines of shears at the entrance and exit of the capillary from the form of these lines in the body of the capillary. This source of error may be eliminated by using different lengths of the same tube but the plan has not been generally used.

3. There is considerable uncertainty in measuring the bore of the tube and when it is not uniform, which usually obtains, in reducing the bore to its proper value.

[1] Émile Mathieu, Comptes Rendus, 1863, Tome 57, p. 320.
[2] F. M. Pedersen, PHYS. REV., October, 1907, pp. 225–254.

4. On account of the smallness of the bore, in the case of metal tubes at least, one cannot be certain that the action of the gas on the wall of the tube does not affect the result.

5. The possibility of distortion of the lines of shear in the body of the tube because of eddies or turbulence is inherent in the capillary tube method.

The effect of all these errors is to give an apparent value of η higher than the actual value. On this account therefore it is probable that the lower values which have been obtained by the capillary tube method are closer to the correct value particularly since there is close agreement among several experimenters whose work was carried out under conditions that were considerably different.

II. *The Damping of the Oscillations of a Suspended Body.*

There are two modifications of this method.

(*a*) A spherical or cylindrical bob attached to a fine suspension has been used as a simple pendulum.

(*b*) The damping of torsional vibrations of spheres and cylinders has been used; in some cases these bodies vibrated in free air and in other cases adjacent to or confined in similar bodies. This was the method used by Tomlinson, Hogg and Reynolds. The formulæ which have been derived to connect the coefficient of viscosity and the damping of the oscillations is approximate and it is difficult to calculate or determine experimentally the allowance that must be made for the suspending parts, or the inner part and edge of the suspended body. The agreement of the results obtained by Tomlinson and Hogg seems to show that they have been very successful in estimating these factors and does much to justify the theories giving rise to the mathematical development of the formulæ which have been used. A criticism of Reynolds's work is given by Hogg in an endeavor to account for the somewhat higher value which was obtained by the former.

Summary.

1. The value of η for air has been determined for a temperature of 20.2° C. to be $1{,}812 \times 10^7$ with a probable error less than .2 per cent.

2. The variation of η with temperature has been found to agree fairly well with the relationship found by S. W. Holman and other experimenters.

3. The presence of water vapor in the air up to 60 per cent. saturation at 20° C. has no effect on η and at saturation the effect is an increase not greater than .3 per cent.

4. The method which has been used for an absolute determination of η for air at normal temperature has been shown to be exceptionally reliable.

In conclusion I desire to acknowledge my indebtedness to Professor A. A. Michelson and the Department of Physics of the University of Chicago for placing at my disposal the facilities necessary for this research, to Professor R. A. Millikan, who suggested the problem and under whose direction the work was done; to Mr. Wm. Gaertner for the loan of the clock-driving apparatus and to the University of Toronto for providing some of the incidental expenses connected with the completion of the work.[1]

[1] A fairly full list of references to the work on the viscosity of gases to 1904 is given by F. G. Reynolds, PHYS. REV., Vol. XVIII., 1904, pp. 427–431, and by F. M. Pederson, PHYS. REV., Vol. 25, 1907, pp. 249–254.

The following may be added to complete the list to date:

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