

Crossed Surface Flat Bands in Three-Dimensional Superconducting Altermagnets

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Superconducting altermagnets have proven to be a promising ground for emergent phenomena, but their study has involved two-dimensional systems. In this work, we investigate three-dimensional d - and g -wave altermagnets with spin-singlet chiral d -wave superconductivity and show the formation of crossed surface flat bands due to the interplay between superconducting and altermagnetic symmetries. We find that these crossed flat bands are topologically protected, appear at zero energy in the surface along z due to the superconducting nodal lines in the xy plane, and their number of corners is determined by the crystal symmetry of altermagnets. We also show that the superconducting nodal lines give rise to Bogoliubov-Fermi surfaces, which then affect the appearance of zero-energy arcs in the surface along x . Moreover, we demonstrate that the crossed flat bands or surface arcs, and Bogoliubov-Fermi surfaces give rise to the coexistence of three distinct dependences of the charge conductance on the normal transparency, hence offering a solid way for their detection and paving the way for realizing higher-dimensional topological phases using altermagnets.

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The discovery of altermagnetism has recently triggered great interest due to its potential for realizing emergent phenomena and spintronic applications [1–4]. Altermagnets (AMs) are characterized by a zero net magnetization as a result of their momentum dependent spin splitting of energy bands and anisotropic spin-polarized Fermi surfaces [2,5–14]. While these properties have proven crucial in the normal state [1], it has been shown that combining AMs with superconductivity offers unprecedented opportunities [4,15]. For instance, altermagnetism has been shown to induce key bulk and surface phenomena [4,15], including anomalous Josephson transport [16–24], topological superconductivity [25–34], exotic superconducting pairs [16,35–47], nonreciprocal transport [48–52], nontrivial light-matter coupling [41,42], and distinct types of stable subgap states [53,54].

The appearance of stable subgap states is of particular relevance because they signal unusual superconductivity protected by symmetry and topology [55–60]. In this regard, altermagnetism has been shown to induce Bogoliubov-Fermi surfaces (BFSs), flat bands, and Andreev bound

states (ABSs), all with direct ties to the altermagnetic and superconducting symmetries [53,54]. While these ideas show the potential of AMs for topological superconducting phases, the existing studies only addressed two-dimensional (2D) systems, leaving unexplored the large family of 3D AMs. One of the most interesting 3D systems is Sr_2RuO_4 , which has very recently been predicted to host altermagnetic order [61,62] as a phase that coexists with a very likely 3D chiral d -wave superconductivity [63–66] probed by nuclear magnetic resonance [68,69]. It is thus natural to wonder about the design and detection of higher-dimensional topological superconducting phases by exploiting the symmetry and topology of 3D superconducting AMs.

In this work, we consider 3D superconducting AMs (Fig. 1) and demonstrate the emergence of crossed surface flat bands as a generic topological phenomenon due to the interplay between superconducting and altermagnetic symmetries. In particular, we focus on d - and g -wave AMs with chiral d -wave superconductivity, which are very likely to appear in Sr_2RuO_4 ; see [61,62] and [63–67]. We discover that the xy -plane nodal lines of chiral d -wave superconductivity ensure that the crossed flat bands appear at zero energy on the [001] surface, while their corners are determined by the altermagnetic nodes. The crossed surface flat bands are topologically protected and represent the higher-dimensional analog of zero-energy ABSs [55,70] in 3D superconducting AMs, thus broadening their initial conception in Weyl superconductors [71] and the idea that

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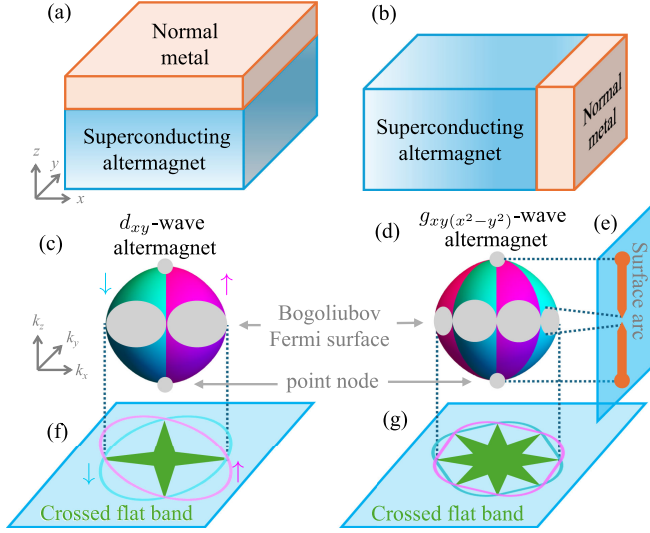


FIG. 1. (a),(b) Sketches of the studied junctions along z (a) and x directions (b), formed by a 3D superconducting altermagnet (light blue) and a normal metal (light red). (c)–(f) Bulk and surface properties of d_{xy} - and $g_{xy(x^2-y^2)}$ -wave altermagnets with chiral d -wave superconductivity. (c),(d) Bulk Fermi volumes for d_{xy} - (c) and $g_{xy(x^2-y^2)}$ -wave (d) altermagnets, where the cyan and magenta colors indicate up and down spins. The line nodes of the Fermi volumes give rise to Bogoliubov-Fermi surfaces (gray) under chiral d -wave superconductivity. (e) A projection of the Fermi volumes on the $[100]$ surface along x leads to surface arc states (orange), whose ends are marked by the point nodes. (f),(g) When projecting the Fermi volumes on the $[001]$ surface along z , crossed flat bands emerge (green) with their corners defined by the nodes of the 2D spin-polarized altermagnetic Fermi surfaces, indicated by cyan and magenta ellipses for down and up spins. The crossed flat bands appear at zero energy, which is ensured by the nodal lines of chiral d wave of superconductivity.

flat bands inherit the shape of the projected Fermi surface [72,73]. Moreover, we find that the superconducting nodal lines induce BFSs, which then modify the formation of arcs on the $[100]$ surface. We further show that the crossed flat bands or arc states, and BFSs coexist and produce very distinctive transparency dependent conductance signals, which can be used for their detection. Our findings establish 3D superconducting AMs as a fertile ground for designing symmetry-protected topological phenomena.

3D superconducting AMs—We consider a 3D superconducting AM modeled by the following Bogoliubov-de Gennes (BdG) Hamiltonian:

$$\hat{H}_{\text{BdG}}^{\alpha}(\mathbf{k}) = \varepsilon(\mathbf{k})\hat{\sigma}_0\hat{\tau}_3 + \hat{H}_{\text{AM}}^{\alpha}(\mathbf{k}) + \hat{H}_{\text{SC}}(\mathbf{k}), \quad (1)$$

where $\varepsilon(\mathbf{k}) = -\mu - 2t \cos k_x - 2t \cos k_y - 2t \cos k_z$, while $\hat{H}_{\text{AM}}^{\alpha}(\mathbf{k})$ and $\hat{H}_{\text{SC}}(\mathbf{k})$ describe the altermagnetic and superconducting orders, with α denoting the type of altermagnetism (see below). Here, $\mathbf{k} = (k_x, k_y, k_z)$, μ is the chemical potential, t is the hopping integral, while $\hat{\sigma}_j$ and $\hat{\tau}_j$ are the

j th Pauli matrices in spin and particle-hole subspaces, respectively. Altermagnetism is modeled by

$$\hat{H}_{\text{AM}}^{\alpha}(\mathbf{k}) = M_k^{\alpha} \hat{\sigma}_3 \hat{\tau}_3, \quad (2)$$

where M_k^{α} characterizes α -wave altermagnetic order with $\alpha = d, g$. We focus on d_{xy} - and $d_{x^2-y^2}$ -wave AMs described by $M_k^{d1} = 2t_{d1} \sin k_x \sin k_y$, $M_k^{d2} = t_{d2}(\cos k_x - \cos k_y)$. We also consider g -wave AMs with $M_k^g = 4t_g \sin k_x \sin k_y (\cos k_x - \cos k_y)$. Thus, t_{d1} , t_{d2} , and t_g represent the strength of d_{xy} -, $d_{x^2-y^2}$ -, and $g_{xy(x^2-y^2)}$ -wave AMs. While $\hat{H}_{\text{AM}}(\mathbf{k})$ is described by an effective 2D model, our findings remain in 3D AMs [74]; see S3 in Supplemental Material (SM) [75]. Moreover, we assume spin-singlet 3D chiral superconductivity modeled by $\hat{H}_{\text{SC}}(\mathbf{k}) = -\Delta \sin k_z [\sin k_x \hat{\tau}_1 + \sin k_y \hat{\tau}_2] \hat{\sigma}_2$, with pair amplitude Δ ; t_{α} is varied within Δ to ensure the coexistence of altermagnetism and superconductivity [81], as shown in S1.2 of [75]. In fact, in S1.2 of [75], we show that the chosen spin-singlet 3D chiral d -wave pairing coexists with altermagnetism by self-consistently solving the gap equation with a second-nearest neighbor attractive interaction [82,83]. Thus, this justifies our choice of 3D chiral d -wave pairing in 3D superconducting AMs and can apply the interorbital pairing of Sr_2RuO_4 [63–66] because the relating symmetry and gap structure are similar to interlayer pairings [64]. Also, Refs. [61,62] predict altermagnetic order in Sr_2RuO_4 , and a very likely 3D chiral d -wave superconductivity in Refs. [63–66]. Although we focus on a particular 3D nodal superconductor, our analysis can be straightforwardly generalized to other superconducting AMs with unconventional superconductivity.

Emergence of crossed surface flat bands—We are interested in unveiling the impact of altermagnetism on superconductivity in a 3D superconducting AM modeled by Eq. (1). For this purpose, it is useful to first inspect the energy bands, which are obtained as

$$E_{\mathbf{k},\pm}^{s,\alpha} = sM_k^{\alpha} \pm \sqrt{\varepsilon^2(\mathbf{k}) + |\bar{\Delta}(\mathbf{k})|^2}, \quad (3)$$

where $s = \pm$ and $\bar{\Delta}(\mathbf{k}) = \Delta \sin k_z [\sin k_x + i \sin k_y]$. These energy bands help understand the appearance of bulk phases. In the absence of AMs, the 3D chiral superconductor has line nodes in the xy plane, but point nodes in the z plane [Figs. 1(c) and 1(d)]. These line and point nodes give rise, respectively, to zero-energy surface flat bands on the $[001]$ surface and to surface arc states on the $[100]$ surface [55,70,72,84]; the shape of the flat bands is determined by that of the projected Fermi surface, with a disk shape for a spherical Fermi surface [84]. Here, we obtain $E_{\mathbf{k},+}^{-,\alpha} = 0$ for $[M_k^{\alpha}]^2 = \varepsilon^2(\mathbf{k}) + |\bar{\Delta}(\mathbf{k})|^2$. This defines the formation of a BFS, and shows that the line nodes of the chiral d -wave superconductor become BFSs due to

altermagnetism, except for the nodal lines of AMs. Interestingly, altermagnetism strongly deforms the surface states of the chiral superconductor, giving rise to *crossed flat bands* and modified surface arcs in the [001] and [100] surfaces [Figs. 1(e)–1(g)]. While the surface arcs are inherited from the chiral superconductor, the crossed flat bands are entirely due to the interplay between altermagnetism and chiral d -wave superconductivity: by projecting the altermagnetic Fermi surfaces, we identify that the altermagnetic nodes (spin-degenerate lines in 3D) define the number of corners of the crossed flat band. Moreover, due to the nodes of the 3D chiral d -wave superconductivity, the crossed flat bands appear at zero energy. This analysis also applies to spin-triplet p_z -wave superconductivity [56]. Thus, the emergence of crossed flat bands is a generic phenomenon resulting from the interplay between altermagnetism and nodal superconductivity and hence tied to the inherent crystal symmetries.

Topological origin of crossed surface flat bands—The symmetries required for the emergence of crossed flat bands also imply that they have a topological origin [56,60,72,73,85,86], which we show here. In 3D superconductors with nodal lines but without altermagnetism, a nonzero 1D winding number can be obtained with the chiral operator by combining the pseudo-time-reversal (pTRS) and particle-hole (PH) symmetries [84], where pTRS is the product of conventional TRS and local gauge transformation. This local gauge transformation is introduced to cancel out the inner phase of the pair potential. In the presence of altermagnetism, the chiral symmetry is broken because of the broken TRS by AMs [1,4]. Still, the Hamiltonian keeps the pseudo-magnetic mirror symmetry (pMMS), which is given by the product of pTRS and mirror symmetry. For d_{xy} - and $d_{x^2-y^2}$ -wave AMs, the pMMSs are given, respectively, by

$$\hat{\Theta}'_k \hat{H}_{\text{BdG}}^{d1}(k_x, k_y, k_z) \hat{\Theta}'_k{}^\dagger = [\hat{H}_{\text{BdG}}^{d1}(-\nu k_x, \nu k_y, k_z)]^* \quad (4)$$

$$\hat{\Theta}'_k \hat{H}_{\text{BdG}}^{d2}(k_x, k_y, k_z) \hat{\Theta}'_k{}^\dagger = [\hat{H}_{\text{BdG}}^{d2}(-\nu k_y, -\nu k_x, k_z)]^*, \quad (5)$$

where $\hat{\Theta}'_k = \hat{U}_{\phi_k}^\dagger \hat{\Theta} \hat{U}_{\phi_k}$ denotes the pMMS, $\hat{\Theta} = i\hat{\sigma}_2 \hat{\tau}_0$ is the conventional TRS, $\hat{U}_{\phi_k} = e^{-i\phi_k \hat{\sigma}_0 \hat{\tau}_3/2}$ is the local gauge transformation, and $\phi_k = \tan^{-1}(\sin k_y / \sin k_x)$ is the phase of the pair potential. Here, $\nu = +1$ and $\nu = -1$ in Eq. (4) indicate pMMSs for d_{xy} -wave AMs in the xz and yz planes, respectively. Also, $\nu = +1$ and $\nu = -1$ in Eq. (5) indicate pMMSs for $d_{x^2-y^2}$ -wave AMs in the diagonal planes on $x = y$ and $x = -y$, respectively. For $g_{xy(x^2-y^2)}$ -wave AMs, the system has both pMMSs given by Eqs. (4) and (5). In addition to the pMMSs, $\hat{H}_{\text{BdG}}^\alpha(\mathbf{k})$ has particle-hole symmetry given by

$$\hat{C} \hat{H}_{\text{BdG}}^\alpha(k_x, k_y, k_z) \hat{C}^\dagger = -[\hat{H}_{\text{BdG}}^\alpha(-k_x, -k_y, -k_z)]^* \quad (6)$$

with $\hat{C} = \hat{\sigma}_0 \hat{\tau}_1$. Then, $\hat{H}_{\text{BdG}}^\alpha(\mathbf{k})$ anticommutes with the chiral operator $\hat{\Gamma}_k = -i\hat{C} \hat{\Theta}'_k$ on the magnetic mirror plane. Concrete forms of $\hat{\Gamma}_k$ on each mirror plane are given in S1.4 of SM [75]. Importantly, these magnetic mirror and chiral symmetries are still available on the smooth surfaces, which are perpendicular to the magnetic mirror planes, allowing for the symmetry-protection of surface states as a unique effect due to altermagnetism and nodal superconductivity. For example, a vertical pMMS is given in Eq. (4) and the corresponding chiral symmetry is accessible on the k_x - and k_y -axes in the surface Brillouin zone. Hence, a winding number can be obtained on the symmetric lines, whose nonzero values signal the appearance of topological phases with topologically protected surface states [75]. Thus, a nonzero winding number along the k_x , k_y axis and/or the diagonal direction implies that the crossed flat bands along the [001] direction discussed in the previous section are topologically protected [Figs. 1(f) and 1(g)]. Also, because of the pMMS on the xz plane, topologically protected zero-energy surface states appear on the [100] surface, which we identified as surface arcs [Fig. 1(e)]. As a result, both surface arcs and crossed flat bands are topologically protected and can be realized in 3D superconducting AMs [87].

Conductance signatures of crossed flat bands—Having shown the emergence of crossed flat bands and surface arcs, as well as their topological origin, here we explore how they can be detected by means of conductance. For this purpose, we couple the 3D superconducting AM to a normal metal, such that we can assess transport along z and x directions [Figs. 1(a) and 1(b)]. We obtain the conductance following the common tunneling Hamiltonian approach based on the Lee-Fisher formula [53,88–90]:

$$\sigma(eV) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} d\tilde{\mathbf{k}} \bar{\sigma}(\tilde{\mathbf{k}}, eV), \quad (7)$$

with $\tilde{\mathbf{k}} = (k_u, k_v)$ for $u = x, y$ and $v = y, z$, while $\bar{\sigma}(\tilde{\mathbf{k}}, eV)$ is the momentum-dependent conductance obtained as

$$\bar{\sigma}(\tilde{\mathbf{k}}, eV) = \text{Tr}' \sum_{i,j=0,1} [\hat{G}_{ii} \hat{V}_{ji}^\dagger \hat{G}_{jj} \hat{V}_{ji} - \hat{G}_{ij} \hat{V}_{ji} \hat{G}_{ij} \hat{V}_{ji}], \quad (8)$$

where we have dropped the momenta for simplicity, and Tr' means that the trace is taken in electron space. Here, $\hat{V}_{10} = -\hat{t}_0 \otimes \hat{\tau}_0$, with $\hat{t}_0$ characterizing the hopping matrix in metallic lead [Figs. 1(a) and 1(b)]. Moreover, $\hat{G}_{nm}$ are Green's functions numerically obtained by using the recursive Green's function method [53,89–93], and the details of this formulation are presented in S2 of SM [75].

We first assess the conductance signatures along z direction [001]; see Fig. 2 [94]. We begin by exploring the momentum-dependent conductance $\bar{\sigma}(\tilde{\mathbf{k}}, eV = 0)$ at zero bias $eV = 0$, which is presented in Figs. 2(a) and 2(c) for

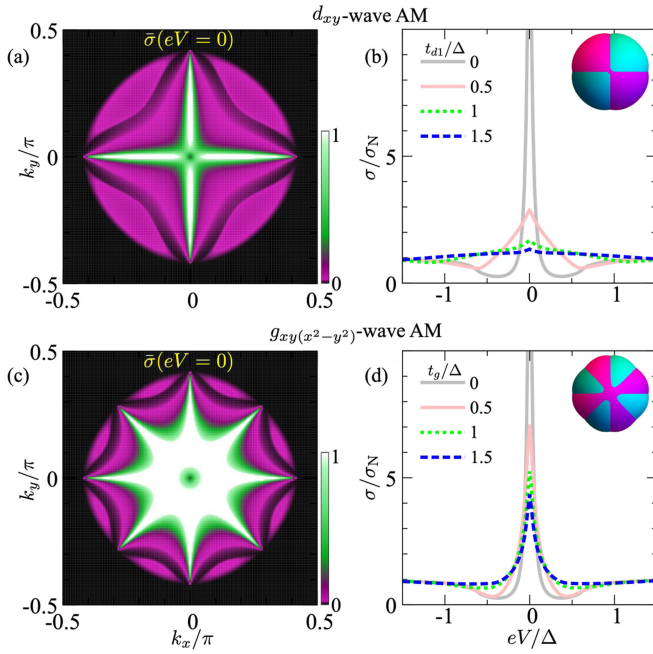


FIG. 2. (a),(c) Zero-bias conductance along the z direction [001] as a function of momenta $k_{x,y}$ for a superconducting AM with d_{xy} -wave (a) and $g_{xy(x^2-y^2)}$ -wave (c) altermagnetism. (b),(d) Normalized total conductance along the z direction as a function of eV for distinct values of the altermagnetic strength t_a . σ_N indicates the conductance in the normal state at $eV = 0$. The insets in (b),(d) indicate the normal state Fermi surfaces projected onto the [001] surface for up (magenta) and down spins (cyan). Parameters: $t_a = \Delta$ (a),(c), $\mu = -4.5t$, $\Delta = 0.01t$, $U_b = 5t$, and $\delta = 0.01\Delta$.

superconducting AMs with d_{xy} - and $g_{xy(x^2-y^2)}$ -wave altermagnetism. The most relevant signature of the momentum-dependent conductance at zero bias is that the strongest intensities come from the contribution of the crossed surface flat bands [Figs. 2(a) and 2(c)], which emerge at the [001] surface as discussed in the previous section. Depending on the type of underlying altermagnetism, the zero-bias conductance $\bar{\sigma}(\tilde{\mathbf{k}}, eV = 0)$ unveils crossed flat bands with different number of corners [Figs. 2(a) and 2(c)], as expected since the corners of the emergent crossed flat bands are determined by the nodes of the AM, where chiral symmetry can be defined. For d_{xy} -wave AM, $\bar{\sigma}(\tilde{\mathbf{k}}, eV = 0)$ reveals four corners [Fig. 2(a)], while eight corners for $g_{xy(x^2-y^2)}$ -wave AM [Fig. 2(c)]. For $d_{x^2-y^2}$ -wave AMs, $\bar{\sigma}(\tilde{\mathbf{k}}, eV = 0)$ also exhibits four corners but with a crossed flat band rotated by $\pi/4$; see End Matter and Fig. 4(a) therein. Moreover, $\bar{\sigma}(\tilde{\mathbf{k}}, eV = 0)$ in Figs. 2(a) and 2(c) acquires lower but nonzero intensities around the crossed flat bands, which arise due to the BFS. This is confirmed by the zero-energy bulk density of states integrated in k_z ; see End Matter and Fig. 5 therein [95]. There also appears a darkish area between the BFS and crossed flat band in Figs. 2(a) and 2(c) originated from a gapped dispersion where quasiparticles attain

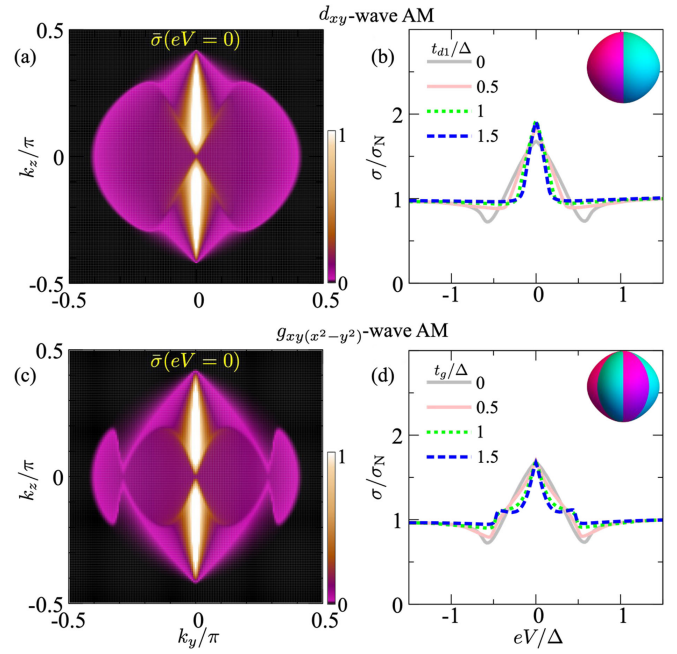


FIG. 3. (a),(c) Zero-bias conductance along the x direction [100] as a function of $k_{y,z}$ for a superconducting AM with d_{xy} -wave (a) and $g_{xy(x^2-y^2)}$ -wave (c) altermagnetism. (b),(d) Normalized total conductance along the x direction [100] as a function of eV for distinct values of the altermagnetic strength t_a . σ_N indicates the conductance in the normal state at $eV = 0$. The insets in (b),(d) indicate the normal state Fermi surfaces projected onto the [100] surface for up (magenta) and down spins (cyan). Parameters: $t_a = \Delta$ (a),(c), $\mu = -4.5t$, $\Delta = 0.01t$, $U_b = 5t$, $\delta = 0.01\Delta$.

vanishing velocity along z ; see S1.3 in SM [75]. Notably, we find that, for the crossed flat bands, BFSs and gapped regions, respectively, we obtain $\bar{\sigma}(\tilde{\mathbf{k}}, eV = 0) \sim a\bar{\sigma}_N^0 + b\bar{\sigma}_N^1 + c\bar{\sigma}_N^2$, with the normal state conductance $\bar{\sigma}_N$, unveiling three distinct types of fundamental power laws that coexist in 3D superconducting AMs [99]; see S2 in SM [75]. The total conductances $\sigma(eV)$ are then shown in Figs. 2(b) and 2(d) as a function of eV for distinct altermagnetic strengths $t_{d1,g}$: $\sigma(eV)$ develops a zero-bias peak whose height decreases as the altermagnetic strength increases; see also Fig. 4(b). The presence of a large zero-bias conductance peak (ZBCP) without any altermagnetism is due to the zero-energy ABSs of chiral d -wave superconductors [55,60,84,101]. Altermagnetism, however, reduces this ZBCP: its reduced value remains even at strong values of altermagnetic strengths [Figs. 2(b) and 2(d)]. While the same reduction occurs for d_{xy} -, $d_{x^2-y^2}$ -, and $g_{xy(x^2-y^2)}$ -wave AMs, the latter exhibits a stronger and well-formed ZBCP; see End Matter and Fig. 8 therein. The signature of the BFS (altermagnetic nodes) appears in the width (height) of the ZBCP of $\sigma(eV)$ [75]. In all cases, the ZBCP constitutes direct evidence of the crossed surface flat bands formed at the [001] surface of 3D superconducting AMs.

Conductance along x direction and signatures of surface arcs—Conductance along x reveals distinct signatures of the 3D superconducting AM, giving rise to direction-dependent transport in these systems. In Figs. 3(a) and 3(c), we present the zero bias momentum-dependent conductance $\bar{\sigma}(\tilde{\mathbf{k}}, eV = 0)$ for superconducting AMs with d_{xy} - and $g_{xy(x^2-y^2)}$ -wave altermagnetism; the End Matter and Fig. 6(a) therein shows $\bar{\sigma}(\tilde{\mathbf{k}}, eV = 0)$ for $d_{x^2-y^2}$ -wave AMs. We see that $\bar{\sigma}(\tilde{\mathbf{k}}, eV = 0)$ develops high intensities around $k_y = 0$ due to the contribution of zero-energy surface arc states formed at the [100] surface [Figs. 3(a) and 3(c)]. As discussed before, these zero-energy surface arcs originate from the pMMS on the xz plane. Interestingly, the surface arc conductance signal is interrupted around $k_z = 0$ due to a reduced contribution coming from the BFSs; see Fig. 1(d); the role of the BFS can indeed be confirmed by the bulk density of states integrated in k_x ; see Fig. 7. For $d_{x^2-y^2}$ -wave AMs in Fig. 6(a), $\bar{\sigma}(\tilde{\mathbf{k}}, eV = 0)$ reveals the formation of spin-split surface arc states, which disperse with $k_{y,z}$ and is smeared by the BFSs. As for transport along z , here we also find that $\bar{\sigma}(\tilde{\mathbf{k}}, eV = 0)$ is given by $2\bar{\sigma}_N^0$, $\bar{\sigma}_N^1$, and $\bar{\sigma}_N^2$ in the presence of zero-energy surface arcs, BFS, and gapped region; see S2.3 in SM [75]. In the case of the total conductance $\sigma(eV)$, Figs. 3(b) and 3(d) show it as a function of eV for distinct strengths of d_{xy} and $g_{xy(x^2-y^2)}$ altermagnetism. It exhibits a broad peaked profile around zero bias without altermagnetism [Figs. 3(b) and 3(d)], resulting from the chiral edge modes [84]. This broad conductance profile sharpens at finite altermagnetism for d_{xy} and $g_{xy(x^2-y^2)}$ -wave AMs due to the change in the dispersion of the chiral edge modes, producing a clear ZBCP [Figs. 3(b) and 3(d)] that stems from the surface arc states [75]. Interestingly, for $d_{x^2-y^2}$ -wave altermagnetism, the resonance in $\sigma(eV = 0)$ splits in half, and the resonance shift depends on the altermagnetic strength [Fig. 6(b)]; this spin-split total conductance originates from the spin-split surface arcs observed in $\bar{\sigma}(\tilde{\mathbf{k}}, eV = 0)$ [Fig. 6(a)]. This behavior is similar to the reported transport in Sr_2RuO_4 [102], supporting the idea that conductance along x is determined by the surface arc states and the BFS.

In conclusion, we have demonstrated that crossed surface flat bands emerge in three-dimensional superconducting altermagnets as a generic topological phenomenon protected by intrinsic crystal symmetries. Also, we have shown that the crossed flat bands can be detected by means of conductance, which depends on the transport direction and can also assess the formation of topological surface arcs and Bogoliubov-Fermi surfaces. We have found that the zero-bias momentum-resolved conductance exhibits three distinct types of fundamental power laws, offering a solid way for detecting the emergent topological phases, e.g., by Doppler shift measurements as those predicted in Refs. [103–106], ARPES [107–110], and quasiparticle interference experiments [111–114], which would require

low transparencies and low temperatures for observing the crossed flat bands; see End Matter. Our results apply to the large family of predicted 3D AMs, such as CrSb [115] and La_2CuO_4 [13], and superconducting UPt_3 [116] and half-Heusler compounds [96]. Notably, our work holds direct relevance in Sr_2RuO_4 , where altermagnetism and chiral d -wave superconductivity are expected to coexist [61–69], being thus a promising material to test our predictions. Our work, therefore, establishes three-dimensional superconducting altermagnets as an outstanding ground for realizing higher-dimensional topological phases, which can also pave the way for future tunneling spectroscopy experiments in three-dimensional superconductors. Based on our topological analysis, we argue that found crossed flat bands are robust against symmetry-preserving perturbations, such as multiorbital spin-orbit coupling and scalar disorder, while symmetry-breaking perturbations are very likely detrimental and require a thorough investigation.

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Data availability—The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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End Matter

Conductance along z -direction with $d_{x^2-y^2}$ -wave AMs— The conductance along z -direction for a superconducting AM junction with $d_{x^2-y^2}$ -wave altermagnetism is presented in Fig. 4. In the case of the momentum dependent conductance at zero-bias voltage $\bar{\sigma}(\tilde{\mathbf{k}}, eV=0)$ in Fig. 4(a), it develops high intensity values along the diagonals $k_x = \pm k_y$ in contrast to the case of d_{xy} -wave altermagnetism having large values at $k_{x,y} = 0$; see Fig. 2(a). These large zero-bias conductance values reveal the emergence of crossed flat bands in the [001] surface of superconducting AMs with $d_{x^2-y^2}$ -wave altermagnetism. As noted in the main text, $\bar{\sigma}(\tilde{\mathbf{k}}, eV=0)$ also reflects the BFS, seen as an area that surrounds the crossed flat band in Fig. 4(a). Between the BFS and the area of the crossed flat band appears a darkish region, which is gapped and quasiparticles acquire vanishing velocity and hence vanishing conductance. Interestingly, $\bar{\sigma}(\tilde{\mathbf{k}}, eV=0)$ acquires a unique dependence on its zero-bias normal state counterpart: $\bar{\sigma}(\tilde{\mathbf{k}}, eV=0)$ is roughly given by $2\bar{\sigma}_N^0$, $\bar{\sigma}_N^1$, and $\bar{\sigma}_N^2$ for the regime with crossed flat bands, BFSs, and gapped region, respectively; see S2 in SM [75]. Also, the

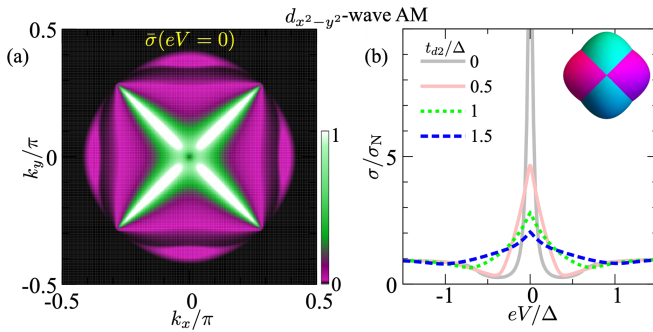


FIG. 4. (a) Zero-bias conductance along the z -direction [001] for a superconducting AM with $d_{x^2-y^2}$ -wave altermagnetism as a function of $k_{x,y}$. (b) Normalized total conductance along the z -direction as a function of eV for distinct values of the altermagnetic strength t_{d2} ; σ_N indicates the conductance in the normal state at $eV = 0$. The inset in (b) shows the normal state Fermi surfaces projected onto the [001] surface for up (magenta) and down spins (cyan). Parameters: $t_{d2} = \Delta$ (a), $\mu = -4.5t$, $\Delta = 0.01t$, $U_b = 5t$, $\delta = 0.01\Delta$.

total conductance $\sigma(eV)$ varies from a very peaked profile at zero bias in the absence of $d_{x^2-y^2}$ -wave altermagnetism to a reduced but visible peak at finite as the altermagnetic strength increases and surpasses the magnitude of the pair potential [Fig. 4(b)]. Thus, the reduced peaked profile manifests the formation of crossed flat bands on the [001] surface.

Bulk density of states along the [001] direction—We have seen that the gapless structure of the energy dispersion in Eq. (3) originates from BFSs. It is thus expected that the density of states (DOS) enhances due to BFSs. To show this, we calculate the bulk DOS as

$$\tilde{D}(\mathbf{k}, E) = -\frac{1}{\pi} \text{Tr}'[\text{Im} \tilde{G}^R(\mathbf{k}, E)], \quad (\text{B1})$$

where Tr' indicates the trace taken in electron space, $\tilde{G}^R(\mathbf{k}, E)$ is the retarded Green's function $\tilde{G}^R(\mathbf{k}, E) = [E + i\delta - \hat{H}_{\text{BdG}}(\mathbf{k})]^{-1}$, with $\hat{H}_{\text{BdG}}(\mathbf{k})$ given by Eq. (1), E and δ are the energy and an infinitesimal positive number. Using Eq. (B1), we obtain the bulk DOS on the [001] direction by integrating for k_z . To unveil the BFS, in Fig. 5 we plot the bulk DOS on the [001] direction at zero energy for d_{xy} -, $d_{x^2-y^2}$ -, and $g_{xy}(x^2-y^2)$ -wave AMs. The most important feature is that the BFS produces the highest intensity in the projected zero-energy DOS; see Fig. 5. Interestingly, the BFSs surround regions with tiny values, which possess corners depending on the type

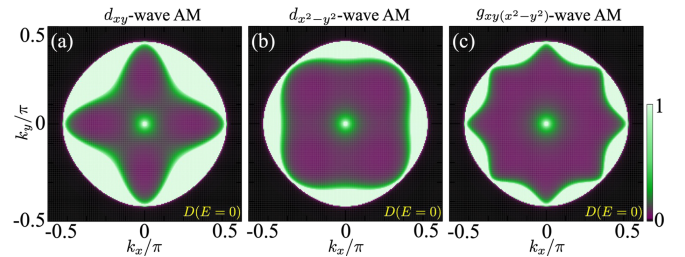


FIG. 5. (a)–(c) Zero-energy bulk DOS integrated in k_z as a function of $k_{x,y}$ for a superconducting AM with d_{xy} - (a), $d_{x^2-y^2}$ - (b), and $g_{xy}(x^2-y^2)$ -wave altermagnetism. Parameters: $t_a = \Delta$, $\mu = -4.5t$, $\Delta = 0.01t$, $U_b = 5t$, $\delta = 0.01\Delta$.

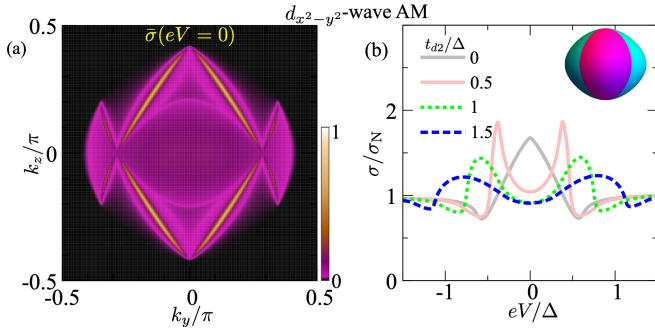


FIG. 6. (a) Zero-bias conductance along the x direction [100] for a superconducting AM with $d_{x^2-y^2}$ -wave altermagnetism as a function of $k_{x,y}$. (b) Normalized total conductance along the x direction [100] as a function of eV for distinct values of the altermagnetic strength t_{d2} . σ_N is the conductance in the normal state at $eV = 0$. The inset in (b) shows the normal state Fermi surfaces projected onto the x direction for up (magenta) and down spins (cyan). Parameters: $t_{d2} = \Delta$ (a), $\mu = -4.5t$, $\Delta = 0.01t$, $U_b = 5t$, $\delta = 0.01\Delta$.

of AM. These inner regions correspond to the formation of the crossed flat bands shown in Figs. 2(a), 2(c), and 4(a). The bright point at $k_{x,y} = 0$ is due to the nodal point where the energy gap closes. The BFS signals in the bulk DOS remain in more elaborated 3D AMs; see Sec. S3 in SM [75].

Conductance along x direction with $d_{x^2-y^2}$ -wave AMs—Conductance along x in a superconducting AM with $d_{x^2-y^2}$ -wave altermagnetism is shown in Fig. 6(a), and we observe the formation of arcs on the [100] surface. Here, the momentum dependent zero-bias conductance $\bar{\sigma}(\tilde{\mathbf{k}}, eV = 0)$ shows that the arcs are spin-split and disperse with momentum, unlike those due to d_{xy} -wave altermagnetism in Fig. 3(a). The region with tiny but nonzero intensities corresponds to BFSs, while even weaker values of $\bar{\sigma}(\tilde{\mathbf{k}}, eV = 0)$ occur in a gapped region; see S1 in SM [75]. Here, we have also verified that $\bar{\sigma}(\tilde{\mathbf{k}}, eV = 0)$ is given by $2\bar{\sigma}_N^0$, $\bar{\sigma}_N^1$, and $\bar{\sigma}_N^2$ for the surface arcs, BFS, and gapped region; see S2 in SM [75]. Moreover, the presence of altermagnetism splits the

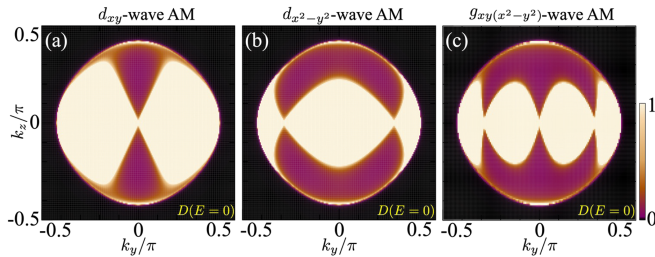


FIG. 7. (a)–(c) Zero-energy bulk DOS integrated in k_x as a function of $k_{y,z}$ for a superconducting AM with d_{xy} - (a), $d_{x^2-y^2}$ - (b), and $g_{xy(x^2-y^2)}$ -wave altermagnetism. Parameters: $t_\alpha = \Delta$, $\mu = -4.5t$, $\Delta = 0.01t$, $U_b = 5t$, $\delta = 0.01\Delta$.

zero-bias total conductance peak of $\sigma(eV)$ in Fig. 6(b), producing two broad resonances around zero bias as a signature of surface arcs.

Bulk density of states along the [100] direction—For the bulk DOS along the [100] direction using Eq. (B1), we find that BFSs appear around $k_z = 0$ for all the cases of altermagnetism [Fig. 7]. Depending on the type of AM, the bulk DOS shows the BFSs coexisting with regions having tiny but nonzero intensities that correspond to the zero-energy surface arcs on the [100] direction [Figs. 3(a), 3(c), and 7(a)]. We also confirm the BFS signatures in proper 3D AMs; see Sec. S2 in SM [75].

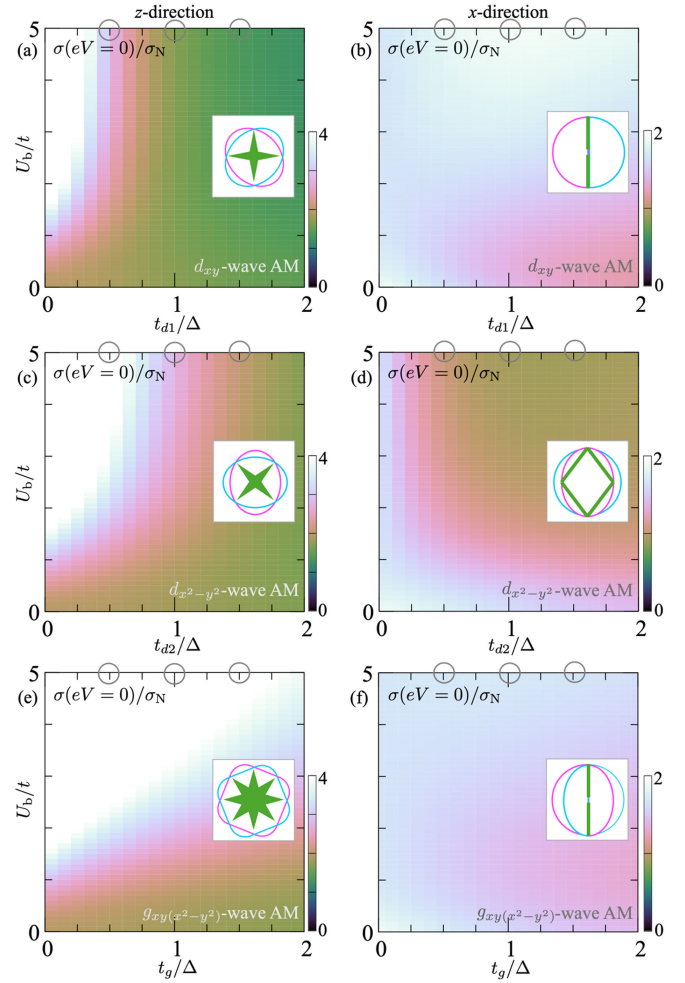


FIG. 8. Normalized zero-bias conductance along z (a),(c),(e) and x (b),(d),(f) directions as a function of U_b and t_α for superconducting AMs with d_{xy} -, $d_{x^2-y^2}$ -, and $g_{xy(x^2-y^2)}$ -wave altermagnetism. Here, σ_N denotes the conductance in the normal state at $eV = 0$, and we calculate it for each t_α and U_b . Insets: crossed flat bands and normal state Fermi surfaces for up (magenta) and down (cyan) spins. The gray circles mark the values of U_b and t_α chosen for Figs. 2 and 3. Parameters: $\mu = -4.5t$, $\Delta = 0.01t$, $\delta = 0.01\Delta$.

Stability of the ZBCP—Having shown the formation of crossed flat bands, surface arcs and BFSs, we now explore the ZBCP $\sigma(eV=0)$ under variations of the altermagnetic strength t_α and barrier potential U_b . This is shown in Fig. 8 for transport along z and x , in both cases for d_{xy} -, $d_{x^2-y^2}$ -, and $g_{xy(x^2-y^2)}$ -wave altermagnetism. For transport along z , the largest zero-bias conductance $\sigma(eV=0)$ is concentrated below $t_\alpha < \Delta$ in the case of d_{xy} - and $d_{x^2-y^2}$ -wave AMs; see Figs. 8(a) and 8(c). For $g_{xy(x^2-y^2)}$ -wave, the ZBCP remains robust even for larger values of t_α ; see Fig. 8(e). These behaviors originate because the crossed flat bands in the respective cases become smaller with the increase of altermagnetic strength t_α . Despite the reduced ZBCPs, their values are still peaked at zero bias for d_{xy} - and $g_{xy(x^2-y^2)}$ -wave AMs, signalling that the crossed flat bands remain for $t_\alpha > \Delta$, as we indeed show in Figs. 2(b)

and 2(d). We also note that the crossed flat bands exist even at $U_b = 0$, since the corners of the crossed flat bands are already formed without any barrier. However, a finite U_b sharpens the conductance, and much larger zero-bias conductance values occur along momenta of the altermagnetic nodal lines. Hence, the visibility of the crossed flat band is enhanced by large U_b , which corresponds to low transparencies. Along the x direction shown in Figs. 8(b), 8(d), and 8(f), the value of $\sigma(eV=0)$ takes values ranging from 1 to 2, and it does not change significantly for variations of U_b and t_α , even though the surface arc signatures partly disappear under BFSs. Also, $\sigma(eV=0)$ becomes smaller as t_{d2} increases because the surface arc states are split by $d_{x^2-y^2}$ -wave altermagnetism [Fig. 6]. The stability of the ZBCP discussed here also persists for 3D AMs; see S3 in SM [75].