


$\Xi_c \rightarrow \Xi$ semileptonic decays: A light cone sum rules view on the experiment-lattice tension

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We present a light-cone QCD sum rule (LCSR) analysis of the semileptonic decays of Ξ_c baryons, focusing on the channels $\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu_\ell$, and $\Xi_c^+ \rightarrow \Xi^0 \ell^+ \nu_\ell$. The transition form factors are calculated within the light-cone quantum chromodynamics (QCD) sum rules framework, using the distribution amplitudes of the heavy Ξ_c baryons. The obtained form factors are then used to compute the differential and total decay widths, as well as the branching fractions. Our numerical results for the branching fractions are $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu_\ell) = (3.73 \pm 1.04)\%$, $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) = (3.59 \pm 1.01)\%$, $\mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 \ell^+ \nu_\ell) = (11.2 \pm 3.25)\%$, and $\mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 \mu^+ \nu_\mu) = (10.8 \pm 3.13)\%$. These results are in good agreement with recent lattice QCD calculations, while being larger than the current experimental measurements and differing from the predictions of other theoretical approaches.

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I. INTRODUCTION

The semileptonic decays of heavy baryons are an important probe of heavy flavor dynamics and the weak interaction. These processes involve a heavy-to-light quark transition accompanied by a lepton–neutrino pair, and they provide valuable information on the underlying Cabibbo-Kobayashi-Maskawa (CKM) matrix elements as well as the structure of the effective weak Hamiltonian. For example, the decays $\Xi_c \rightarrow \Xi \ell^+ \nu_\ell$ are governed at the quark level by the $c \rightarrow s$ transition and are sensitive to the CKM element V_{cs} . These modes also serve as a testing ground for heavy-quark symmetry in the baryon sector. Since the leptonic current in semileptonic decays is well understood theoretically, the primary uncertainties reside in the hadronic transition matrix elements. Accurate knowledge of the transition form factors is therefore crucial for interpreting current and future measurements of heavy baryon semileptonic decays.

Experimentally, charmed antitriplet baryons (such as Λ_c^+ , Ξ_c^+ , and Ξ_c^0) have recently become accessible to precision studies, and significant progress has been reported. The absolute branching fractions of Ξ_c^0 and Ξ_c^+ were not directly

measured; instead, they were determined relative to reference channels such as $\Xi_c^0 \rightarrow \Xi^- \pi^+$ and $\Xi_c^+ \rightarrow \Xi^- \pi^+ \pi^+$. Recently the Belle Collaboration reported the first measurement of the absolute branching fraction for $\Xi_c^0 \rightarrow \Xi^- \pi^+$, as $(1.80 \pm 0.50 \pm 0.14)\%$ [1].

Despite these recent experimental efforts, a notable discrepancy has emerged concerning the branching fractions of $\Xi_c \rightarrow \Xi \ell^+ \nu_\ell$ decays. Recent measurements by the Belle Collaboration reported branching fractions of $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (1.31 \pm 0.04 \pm 0.07 \pm 0.38)\%$ and $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) = (1.27 \pm 0.06 \pm 0.10 \pm 0.37)\%$ [2]. The ALICE Collaboration’s measurement for $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e)$ was $(2.48 \pm 0.25 \pm 0.40 \pm 0.72)\%$ [3,4], while the Particle Data Group (PDG) reports an average of $(1.05 \pm 0.20)\%$ [5]. These experimental values are considerably lower than expectations based on heavy-quark symmetry and flavor symmetry [4]. With more high-luminosity data expected from facilities like BESIII, LHCb, and Belle II, increasingly precise determinations of such branching ratios and decay distributions will become available.

This tension also extends to theoretical predictions. Various models—including lattice QCD [4], relativistic quark models [6], and flavor SU(3) approaches [7,8], generally predict higher branching fractions than the current experimental average. For instance, a recent lattice QCD calculation predicts $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = 3.58(12)\%$ [4], which explicitly notes that this is “much higher than the more recent experimental results” but “reasonably close to the expectation from approximate SU(3) flavor symmetry.” Moreover, the first lattice QCD study [9] also obtained a

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value larger than the experimental measurement, though smaller than the most recent lattice calculation. These discrepancies highlight the need for further theoretical and experimental investigations to improve our understanding of the nonperturbative aspects of charmed baryon decays.

On the theoretical side, the main challenge lies in calculating the form factors, which encapsulate the non-perturbative QCD effects. Several approaches have been used to study semileptonic heavy baryon decays. These include symmetry-based treatments using $SU(3)_f$ flavor symmetry [7,10,11], constituent quark model calculations [8,12–14], lattice QCD simulations [4,9,15], and QCD sum rule techniques [16–19]. Each framework comes with its own advantages and limitations. For instance, $SU(3)$ flavor symmetry can relate different decay channels, but being an approximate symmetry, it inherently induces on the order of 10% theoretical uncertainty in decay amplitudes. Lattice QCD provides first-principles computations but is computationally intensive and has only recently begun to tackle charmed baryon form factors. Light-cone QCD sum rules offer a complementary approach: by expanding a suitable correlator near the light-cone, one can express the hadronic form factors in terms of universal baryonic light-cone distribution amplitudes (DAs) and perturbatively calculable hard kernels. This technique has been successfully applied to various problems in hadron physics, particularly in heavy-to-light transitions. For example, LCSR has been used to analyze the semileptonic decays of Ξ_c baryons [17–19], where the sum rules were formulated using the light-cone DAs of the final-state light baryons.

In this work, we perform an independent calculation of the transition form factors for the semileptonic decays $\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu_\ell$, and $\Xi_c^+ \rightarrow \Xi^0 \ell^+ \nu_\ell$ using the light-cone QCD sum rule approach. By using the distribution amplitudes of the initial charmed baryon Ξ_c , this strategy benefits from heavy quark symmetry, where the charm quark acts as a static color source at leading order, allowing a more controlled description of the baryon's light-quark structure. All six relevant form factors for each transition are computed and used to predict decay widths and branching ratios.

The paper is organized as follows. In Sec. II, we derive the light-cone sum rules for the six transition form factors. Section III presents the numerical analysis, including predictions for decay widths and branching ratios, and compares our findings with results from the literature. Section IV summarizes our main findings and highlights their implications for current and future experimental efforts.

II. CALCULATION OF THE BARYONIC FORM FACTORS FOR $\Xi_c \rightarrow \Xi$ TRANSITION

In this section, we derive the light-cone sum rules for the transition form factors that describe the semileptonic

decays of Ξ_c charmed baryons into light baryons, namely, $\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu_\ell$ and $\Xi_c^+ \rightarrow \Xi^0 \ell^+ \nu_\ell$.

These semileptonic decays are induced by the $c \rightarrow s$ transition and the matrix element can be written as

$$M = \frac{G_F}{\sqrt{2}} V_{cs} \langle \Xi(p') | \bar{c} \gamma_\mu (1 - \gamma_5) s | \Xi_c(p) \rangle \bar{\ell} \gamma^\mu (1 - \gamma_5) \nu, \quad (1)$$

where G_F is the Fermi constant, and V_{cs} is the element of the CKM matrix.

The hadronic matrix element $\langle \Xi(p') | \bar{c} \gamma_\mu (1 - \gamma_5) s | \Xi_c(p) \rangle$ is parametrized in terms of six independent form factors as follows:

$$\begin{aligned} & \langle \Xi(p') | \bar{c} \gamma_\mu (1 - \gamma_5) s | \Xi_c(p) \rangle \\ &= \bar{u}_\Xi(p') \left\{ f_1 \gamma_\mu - i \frac{\sigma_{\mu\nu} q^\nu}{m_{\Xi_c}} f_2 + \frac{f_3 q_\mu}{m_{\Xi_c}} \right. \\ & \quad \left. - g_1 \gamma_\mu \gamma_5 + i \frac{\sigma_{\mu\nu} q^\nu}{m_{\Xi_c}} \gamma_5 g_2 - \frac{g_3 q_\mu}{m_{\Xi_c}} \gamma_5 \right\} u_{\Xi_c}(p). \quad (2) \end{aligned}$$

Here $q = p - p'$ is the momentum transfer, and $f_i(q^2)$ and $g_i(q^2)$ (for $i = 1, 2, 3$) are the vector and axial-vector form factors, respectively.

Since the form factors belong to the nonperturbative sector of QCD, their calculation requires a nonperturbative approach. Among the available methods, QCD sum rules [20,21] hold an exceptional place, as they are firmly grounded in the fundamental QCD Lagrangian and offer a systematic framework for studying hadronic properties. In the present work, we use the light-cone QCD sum rule approach to calculate these form factors.

The main object of this method is the correlation function, which involves the time-ordered product of two operators: the interpolating current for the final-state baryon and the weak transition current responsible for the $c \rightarrow s$ transition. This correlation function is evaluated between the vacuum and the baryon state (in our case, the heavy baryon Ξ_c)

$$\begin{aligned} \Pi_\mu(p, q) &= i \int d^4x e^{ip'x} \langle 0 | T \{ \eta_\Xi(x) \bar{s}(0) \gamma_\mu (1 - \gamma_5) c(0) \} \\ & \quad \times | \Xi_c(p) \rangle, \quad (3) \end{aligned}$$

where η_Ξ is the interpolating current corresponding to the light baryon Ξ . The explicit form of this interpolating current is given by [22]

$$\eta_\Xi(x) = 2\epsilon^{abc} \sum_{\ell=1}^2 (s^a A^\ell d^b) B^\ell s^c, \quad (4)$$

where a, b, c are color indices, and the matrices A^ℓ and B^ℓ correspond to

$$A^1 = C, \quad B^1 = \gamma_5, \quad A^2 = C\gamma_5, \quad B^2 = \beta\mathbb{1},$$

with β being an arbitrary parameter and C is the charge conjugation operator. The choice $\beta = -1$ corresponds to the Ioffe current [23].

To derive the sum rules for the relevant form factors, we evaluate the correlation function in two different kinematic regions: in terms of hadronic degrees of freedom on one side, and in terms of quark and gluon fields using the operator product expansion (OPE) on the other. Matching the two representations through the dispersion relation allows us to extract the desired sum rules. We begin by computing the hadronic representation of the correlation function. This is achieved by inserting a complete set of intermediate hadronic states with the same quantum numbers as the interpolating current η_{Ξ} , and isolating the contribution from the ground-state Ξ -baryon. The result is

$$\begin{aligned} \Pi_{\mu}(p, q) &= \frac{\langle 0 | \eta_{\Xi}(p') | \Xi(p', s') \rangle \langle \Xi(p', s') | J_{\mu} | \Xi_c(p, s) \rangle}{m_{\Xi}^2 - p'^2} \\ &+ \dots, \end{aligned} \quad (5)$$

where the ellipsis denotes contributions from higher resonances and continuum states. Using the definition of the matrix element

$$\langle 0 | \eta_{\Xi}(p') | \Xi(p', s') \rangle = \lambda_{\Xi} u(p', s'), \quad (6)$$

together with the decomposition of the transition matrix element given in Eq. (2), and summing over the spins of the final-state baryon, we derive the hadronic representation of the correlation function

$$\begin{aligned} \Pi_{\mu}^{\text{had}} &= \frac{\lambda_{\Xi}}{m_{\Xi}^2 - p'^2} (\not{p}' + m_{\Xi}) \left\{ f_1 \gamma_{\mu} - i \frac{\sigma_{\mu\nu} q^{\nu}}{m_{\Xi_c}} f_2 + \frac{f_3 q_{\mu}}{m_{\Xi_c}} \right. \\ &\quad \left. - \left[g_1 \gamma_{\mu} - i \frac{\sigma_{\mu\nu} q^{\nu}}{m_{\Xi_c}} g_2 + \frac{g_3 q_{\mu}}{m_{\Xi_c}} \right] \gamma_5 \right\} u(p, s) + \dots, \end{aligned} \quad (7)$$

where λ_{Ξ} is the residue of the Ξ baryon. In the heavy quark limit, it is convenient to express the momentum of the initial heavy baryon as $p_{\mu} = m_{\Xi_c} v_{\mu}$, where v_{μ} is the four-velocity of the heavy baryon. Additionally, the heavy baryon spinor satisfies the relation $\not{v} u(v) = u(v)$.

To proceed further, we decompose the correlation function Π_{μ} into Lorentz-invariant amplitudes (invariant functions), each multiplying a distinct Dirac structure

$$\begin{aligned} \Pi_{\mu} &= \Pi_1 v_{\mu} + \Pi_2 v_{\mu} \gamma_5 + \Pi_3 \gamma_{\mu} + \Pi_4 \gamma_{\mu} \gamma_5 + \Pi_5 q_{\mu} + \Pi_6 q_{\mu} \gamma_5 \\ &+ \dots \end{aligned} \quad (8)$$

Below, we present the QCD-side expression for the correlation function corresponding to the $\Xi_c \rightarrow \Xi$

transition. The QCD representation is obtained from Eq. (1) by applying Wick's theorem, which allows us to contract quark fields into propagators. This leads to the following expression:

$$\begin{aligned} \Pi_{\mu} &= 2e^{abc} \sum_{\ell=1}^2 \int d^4 x e^{i p' x} (A^{\ell})_{\alpha\beta} (B^{\ell})_{\rho\gamma} (\gamma_{\mu} (1 - \gamma_5))_{h\phi} \\ &\times \{ S_{\gamma h} \langle 0 | s_{\alpha}^a(x) d_{\beta}^b(x) c_{\phi}^c(0) | \Xi_c \rangle \\ &+ S_{ah} \langle 0 | s_{\gamma}^a(x) d_{\beta}^b(x) c_{\phi}^c(0) | \Xi_c \rangle \}, \end{aligned} \quad (9)$$

where S is the strange quark propagator. As seen from Eq. (9), the evaluation of the correlation function requires knowledge of the matrix element

$$e^{abc} \langle 0 | s_{\alpha}^a(x) d_{\beta}^b(x) c_{\gamma}^c(0) | \Xi_c(v) \rangle.$$

which is expressed in terms of the light-cone distribution amplitudes of the Ξ_c baryon [24] in the following way:

$$\begin{aligned} e^{abc} \langle 0 | s_{\alpha}^a(t_1 n) d_{\beta}^b(t_2 n) h_{\gamma}^c(0) | \Xi_c(v) \rangle \\ = \sum_{j=1}^4 \mathcal{A}_j (\Gamma_j)_{\alpha\beta} (u_j(v))_{\gamma}, \end{aligned} \quad (10)$$

where h_{γ}^c is the heavy quark field in HQET, Γ_j are Dirac structures, \mathcal{A}_j represents distribution amplitudes, and $u_j(v)$ are spinors describing the light degrees of freedom in the heavy baryon.

We emphasize that these DAs are derived within the framework of heavy quark effective theory (HQET). The relation between the heavy baryon states in full QCD and those in HQET is given by

$$| \Xi_c(p) \rangle = \sqrt{m_{\Xi_c}} | \Xi_c(v) \rangle,$$

where v is the four-velocity of the heavy baryon.

The Dirac structures Γ_j and their associated coefficients \mathcal{A}_j in Eq. (10) are given as follows:

$$\begin{aligned} \mathcal{A}_1 &= \frac{1}{8} f^{(2)} \psi_2(t_1, t_2), & \Gamma_1 &= \not{v} \gamma_5 C^{-1}, \\ \mathcal{A}_2 &= -\frac{1}{8} f^{(1)} \psi_3^{\sigma}(t_1, t_2), & \Gamma_2 &= i \sigma_{\mu\nu} \bar{n}^{\mu} n^{\nu} \gamma_5 C^{-1}, \\ \mathcal{A}_3 &= \frac{1}{4} f^{(1)} \psi_3^s(t_1, t_2), & \Gamma_3 &= \gamma_5 C^{-1}, \\ \mathcal{A}_4 &= \frac{1}{8} f^{(2)} \psi_4(t_1, t_2), & \Gamma_4 &= \not{v} \gamma_5 C^{-1}. \end{aligned} \quad (11)$$

The functions ψ_2 , ψ_3^s , ψ_3^{σ} , and ψ_4 are light-cone distribution amplitudes of twist 2, 3, and 4, respectively.

The light-cone vectors n_μ and \bar{n}_μ are defined as

$$n_\mu = \frac{x_\mu}{vx}, \quad (12)$$

$$\bar{n}_\mu = 2v_\mu - n_\mu. \quad (13)$$

The DAs $\psi(t_1, t_2)$ are related to their momentum-space counterparts through a double Fourier transform

$$\psi(t_1, t_2) = \int_0^\infty \omega d\omega \int_0^1 du e^{-i\omega(t_1\bar{u}+t_2u)} \psi(\omega, u), \quad (14)$$

where ω is the total light-cone momentum of the two light quarks, and $\bar{u} = 1 - u$.

In our case, $t_1 = t_2 = vx$, which simplifies the expression as follows:

$$\psi(t_1, t_2) = \int_0^\infty \omega d\omega \int_0^1 du e^{-i\omega vx} \psi(\omega, u). \quad (15)$$

The explicit expressions of these DAs $\psi_2(\omega, u)$, $\psi_3^s(\omega, u)$, $\psi_3^g(\omega, u)$, $\psi_4(\omega, u)$ for the Ξ_c baryon have been derived in [24], and serve as inputs in our calculation.

Combining Eqs. (9) and (10), the QCD-side expression for the correlation function takes the form

$$\begin{aligned} \Pi_\mu^{\text{QCD}} &= 2i \int d^4x \int_0^1 du \int_0^\infty \omega d\omega e^{i(p'-\omega v)x} \sum_{\ell=1}^2 \sum_{j=1}^4 \mathcal{A}_j \\ &\times \{ \text{Tr}[\Gamma_j A^\ell] B^\ell S\gamma_\mu (1 - \gamma_5) \\ &+ [B^\ell \Gamma_j^T A^{\ell T} S\gamma_\mu (1 - \gamma_5)] \} u(v). \end{aligned} \quad (16)$$

After performing integration over x , the correlation function is expressed in terms of the DAs and propagators in momentum space.

To extract the form factors, we match the coefficients of the relevant Lorentz structures appearing in both the hadronic and QCD representation of the correlation function. We choose the following structures:

$$v_\mu, v_\mu \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, q_\mu, \quad \text{and} \quad q_\mu \gamma_5.$$

Matching the coefficients of these Lorentz structures allows us to derive the sum rules for the transition form factors as

$$\begin{aligned} \frac{2\lambda_\Xi m_{\Xi_c}}{m_\Xi^2 - p'^2} f_1 &= \Pi_1^{\text{QCD}} \\ -\frac{2\lambda_\Xi m_{\Xi_c}}{m_\Xi^2 - p'^2} g_1 &= \Pi_2^{\text{QCD}} \\ \frac{\lambda_\Xi}{m_\Xi^2 - p'^2} \left[(m_\Xi - m_{\Xi_c}) \left(f_1 + \frac{f_2}{m_{\Xi_c}} (m_\Xi + m_{\Xi_c}) \right) \right] &= \Pi_3^{\text{QCD}} \\ -\frac{\lambda_\Xi}{m_\Xi^2 - p'^2} \left[(m_\Xi + m_{\Xi_c}) \left(g_1 + \frac{g_2}{m_{\Xi_c}} (m_\Xi - m_{\Xi_c}) \right) \right] &= \Pi_4^{\text{QCD}} \\ \frac{\lambda_\Xi}{m_\Xi^2 - p'^2} \left[-2f_1 - \frac{f_2}{m_{\Xi_c}} (m_\Xi + m_{\Xi_c}) + \frac{f_3}{m_{\Xi_c}} (m_\Xi + m_{\Xi_c}) \right] &= \Pi_5^{\text{QCD}} \\ \frac{\lambda_\Xi}{m_\Xi^2 - p'^2} \left[2g_1 + \frac{g_2}{m_{\Xi_c}} (m_\Xi - m_{\Xi_c}) - \frac{g_3}{m_{\Xi_c}} (m_\Xi - m_{\Xi_c}) \right] &= \Pi_6^{\text{QCD}} \end{aligned} \quad (17)$$

The invariant amplitudes Π_i^{QCD} can be expressed in terms of dispersion integrals over the spectral densities as

$$\Pi_i^{\text{QCD}} = \int_0^1 du \int_0^\infty d\sigma \sigma \left\{ \frac{\rho_i^{(1)}}{\bar{\sigma}\Delta} + \frac{\rho_i^{(2)}}{\bar{\sigma}^2\Delta^2} + \frac{\rho_i^{(3)}}{\bar{\sigma}^3\Delta^3} \right\}, \quad (18)$$

with

$$\begin{aligned} \Delta &= p'^2 - s(\sigma), & \sigma &= \frac{\omega}{m_\Xi}, \\ s(\sigma) &= \frac{m_s^2 + m_\Xi^2 \bar{\sigma} \sigma - \sigma q^2}{\bar{\sigma}}, & \bar{\sigma} &= 1 - \sigma. \end{aligned} \quad (19)$$

To suppress the contributions of higher resonances and the continuum, we apply a Borel transformation with respect to p'^2 . Matching the hadronic and QCD sides of each

invariant amplitude yields the desired sum rules for the transition form factors.

The Borel transformation and continuum subtraction are performed using the following master formula [25,26]:

$$\begin{aligned} &\int_0^{s_0} ds \frac{I_n}{(p'^2 - s)^n} \\ &= \int_0^{\sigma_0} d\sigma (-1)^n \frac{e^{-s(\sigma)/M^2} I_n}{(n-1)!(M^2)^{n-1}} \\ &+ \left[\frac{(-1)^n}{(n-1)!} e^{-s/M^2} \sum_{j=1}^{n-1} \frac{1}{(M^2)^{n-j-1}} \frac{1}{s^j} \left(\frac{d}{d\sigma} \frac{1}{s'} \right)^{j-1} I_n \right]_{\sigma=\sigma_0}, \end{aligned} \quad (20)$$

with

$$s' = \frac{ds}{d\sigma},$$

$$\left(\frac{d}{d\sigma} \frac{1}{s'}\right)^n I_n \Rightarrow \text{nested derivatives acting on } I_n \quad \text{and}$$

$$I_n = \frac{\sigma \rho^{(n)}}{\bar{\sigma}^n}.$$

Here, σ_0 is the solution of the equation $s(\omega) = s_0$, with s_0 denoting the continuum threshold. Since the Borel-transformed invariant functions Π_i have lengthy expressions, we do not present them explicitly here.

At the end of this section, we provide the expressions for the differential decay width of the semileptonic transition. To this end, we employ the helicity amplitude formalism (see [27]).

These amplitudes are conveniently calculated in the rest frame of the initial heavy baryon, with the z -axis aligned along the momentum of the off-shell W boson.

The vector current helicity amplitudes are given as

$$\begin{aligned} H_{+\frac{1}{2},t}^V &= \frac{\sqrt{Q_+}}{\sqrt{q^2}} \left(m_- f_1^V + \frac{q^2}{m_\Xi} f_3^V \right), \\ H_{+\frac{1}{2},+1}^V &= \sqrt{2Q_-} \left(f_1^V + \frac{m_+}{m_\Xi} f_2^V \right), \\ H_{+\frac{1}{2},0}^V &= \frac{\sqrt{Q_-}}{\sqrt{q^2}} \left(m_+ f_1^V + \frac{q^2}{m_\Xi} f_2^V \right), \end{aligned} \quad (21)$$

where $m_\pm = m_{\Xi_c} \pm m_\Xi$, and $Q_\pm = m_\pm^2 - q^2$.

The axial-vector helicity amplitudes are obtained from the vector ones through the substitutions

$$\begin{aligned} H_{+\frac{1}{2},t}^A &= H_{+\frac{1}{2},t}^V |_{Q_+ \rightarrow Q_-, m_- \rightarrow m_+, f_i^V \rightarrow g_i^V, f_3^V \rightarrow -g_3^V}, \\ H_{+\frac{1}{2},+1}^A &= H_{+\frac{1}{2},+1}^V |_{Q_- \rightarrow Q_+, m_+ \rightarrow m_-, f_1^V \rightarrow g_1^V, f_2^V \rightarrow -g_2^V}, \\ H_{+\frac{1}{2},0}^A &= H_{+\frac{1}{2},0}^V |_{Q_- \rightarrow Q_+, m_+ \rightarrow m_-, f_1^V \rightarrow g_1^V, f_2^V \rightarrow -g_2^V}, \end{aligned} \quad (22)$$

From parity considerations, the helicity amplitudes satisfy the relation

$$H_{-\lambda, -\lambda_W}^{V(A)} = \pm H_{\lambda, \lambda_W}^{V(A)},$$

where the first index refers to the helicity of the final-state (daughter) baryon, and the second to the W boson.

After standard calculations, the differential decay width takes the form

$$\frac{d\Gamma}{dq^2} = \Gamma_0 \frac{(q^2 - m_\ell^2)^2}{m_\Xi^2 q^2} |\vec{p}| H_{\text{tot}}, \quad (23)$$

where the total helicity amplitude is

$$H_{\text{tot}} = H_U + H_L + \frac{m_\ell^2}{2q^2} (H_U + H_L + 3H_S), \quad (24)$$

where

$$\begin{aligned} H_U &= |H_{+\frac{1}{2},+1}|^2 + |H_{-\frac{1}{2},-1}|^2, \\ H_L &= |H_{+\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2, \\ H_S &= |H_{+\frac{1}{2},t}|^2 + |H_{-\frac{1}{2},t}|^2. \end{aligned} \quad (25)$$

The full helicity amplitude is defined as the difference between vector and axial-vector components

$$H_{\lambda, \lambda_W} = H_{\lambda, \lambda_W}^V - H_{\lambda, \lambda_W}^A, \quad (26)$$

and the overall normalization factor Γ_0 is

$$\Gamma_0 = \frac{G_F^2 |V_{cs}|^2 m_{\Xi_c}^5}{192\pi^3}. \quad (27)$$

The three-momentum magnitude of the final-state baryon is

$$|\vec{p}| = \frac{1}{2m_{\Xi_c}} \lambda^{1/2}(m_{\Xi_c}^2, m_\Xi^2, q^2), \quad (28)$$

where $\lambda(a, b, c)$ is the usual Källén function

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc. \quad (29)$$

III. NUMERICAL ANALYSIS

In this section, we present the numerical analysis of the light-cone sum rules for the transition form factors. Using the derived form factors, we estimate the branching ratios for the semileptonic decays $\Xi_c \rightarrow \Xi \ell \nu$. The central non-perturbative inputs in the LCSR framework are the distribution amplitudes of the initial heavy baryon. As mentioned earlier, we employ the Ξ_c baryon DAs derived in [24], whose explicit forms are given by

$$\begin{aligned} \psi_2(u, \omega) &= \omega^2 u \bar{u} \sum_{i=0}^2 \frac{a_i C_i^{3/2}(2u-1)}{\epsilon_i^4 |C_i^{3/2}|^2} e^{-\omega/\epsilon_i}, \\ \psi_3^{(\sigma, s)}(u, \omega) &= \frac{\omega}{2} \sum_{i=0}^2 \frac{a_i C_i^{1/2}(2u-1)}{\epsilon_i^3 |C_i^{3/2}|^2} e^{-\omega/\epsilon_i}, \\ \psi_4(u, \omega) &= \sum_{i=0}^2 \frac{a_i C_i^{1/2}(2u-1)}{\epsilon_i^2 |C_i^{1/2}|^2} e^{-\omega/\epsilon_i}, \end{aligned} \quad (30)$$

where $C_i^\ell(2u-1)$ are Gegenbauer polynomials, and the normalization factors are defined as

$$|C_i^\ell|^2 = \int_0^1 du |C_i^\ell(2u-1)|^2. \quad (31)$$

The numerical values of the shape parameters a_i and ϵ_i are taken from [24]. The normalization constants $f^{(1)}$ and $f^{(2)}$ which appear in the definitions of a_i , are given by [24]

$$f^{(1)} = f^{(2)} = (2.23 \pm 0.35) \times 10^{-2} \text{ GeV}^3.$$

Additional input parameters used in the numerical analysis are adopted from the Particle Data Group (PDG) [5] as summarized in Table I.

In addition to the above inputs, the sum rule expressions involve three auxiliary parameters

- (i) the Borel mass parameter M^2 ,
- (ii) the continuum threshold s_0 , and
- (iii) the mixing parameter β , which appears in the interpolating current. In numerical calculations, we used the Ioffe current, i.e., $\beta = -1$.

A detailed discussion of the stability of the results with respect to the variation of these auxiliary parameters is provided below.

The continuum threshold s_0 is chosen such that the mass obtained from the two-point QCD sum rule reproduces the experimental baryon mass within 10% accuracy. Our numerical analysis shows that this condition is satisfied when

$$s_0 = (3.5 \pm 0.5) \text{ GeV}^2.$$

The working region of the Borel parameter M^2 is determined by imposing that both the higher-twist corrections and the continuum contributions remain subdominant compared to the leading-twist term. This condition ensures that the dominant contribution to the sum rule arises from the lowest-twist DA.

Under these criteria, our numerical analysis yields the following working window for the Borel mass parameter

$$2.0 \text{ GeV}^2 \leq M^2 \leq 3.0 \text{ GeV}^2.$$

Once the working regions of the auxiliary parameters M^2 and s_0 are established, we proceed to analyze the

TABLE I. Input parameters used in the numerical analysis (values from [5]).

$ V_{cs} $	0.975 ± 0.006
$m_c(\bar{m}_c)$	$1.273 \pm 0.046 \text{ GeV}$
$m_{\Xi_c^0}$	$2470.44 \pm 0.28 \text{ MeV}$
$m_{\Xi_c^+}$	$2467.71 \pm 0.23 \text{ MeV}$
m_{Ξ^-}	$1321.71 \pm 0.07 \text{ MeV}$
m_{Ξ^0}	$1314.86 \pm 0.20 \text{ MeV}$
$\tau_{\Xi_c^0}$	$150.4 \pm 2.8 \text{ fs}$
$\tau_{\Xi_c^+}$	$453 \pm 5 \text{ fs}$

q^2 -dependence of the form factors. Note that the LCSR predictions are valid only in the low- q^2 region and must be extrapolated to cover the full physical kinematic range

$$m_\rho^2 \leq q^2 \leq (m_{\Xi_c^-} - m_{\Xi})^2.$$

In particular, the reliability of the sum rules deteriorates at higher q^2 values. The range where the LCSR calculation remains reliable is

$$q^2 \leq 0.5 \text{ GeV}^2.$$

To extrapolate the LCSR predictions to the entire kinematic range, we use the model-independent z -series expansion (Boyd-Grinstein-Lebed or BGL approach) [28].

The conformal mapping is defined as

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad (32)$$

where

$$t_+ = (m_{\Xi_c^-} + m_{\Xi})^2, \quad t_0 = (m_{\Xi_c^-} - m_{\Xi})^2.$$

We find that the LCSR predictions for the form factors are best reproduced by the following fit function:

$$f(q^2) = \frac{1}{1 - q^2/m_{\text{pole}}^2} \{a_0^f + a_1^f z(q^2) + a_2^f z^2(q^2)\}, \quad (33)$$

The pole masses used in the fits are

$$m_{\text{pole}} = \begin{cases} 2.112 \text{ GeV} & \text{for } f_1, f_2, \\ 2.535 \text{ GeV} & \text{for } g_1, g_2, \\ 2.317 \text{ GeV} & \text{for } f_3, \\ 1.969 \text{ GeV} & \text{for } g_3. \end{cases}$$

The fit parameters a_0^f, a_1^f, a_2^f for each form factor are extracted by performing a least-squares match of the parametrization to the LCSR predictions in the region $0 \leq q^2 \leq 0.5 \text{ GeV}^2$. The resulting fits are employed to extend the form factors to the entire kinematic range, enabling reliable decay width and branching ratio predictions. Table II summarizes the form factor values at $q^2 = 0$ for the $\Xi_c^- \rightarrow \Xi \ell^+ \nu_\ell$ transitions.

TABLE II. Form factors f_i and g_i at $q^2 = 0$ for the $\Xi_c^0 \rightarrow \Xi \ell^+ \nu_\ell$ transition.

f_1	0.84 ± 0.11
f_2	0.49 ± 0.06
f_3	-0.35 ± 0.11
g_1	0.84 ± 0.11
g_2	0.49 ± 0.06
g_3	-0.42 ± 0.08

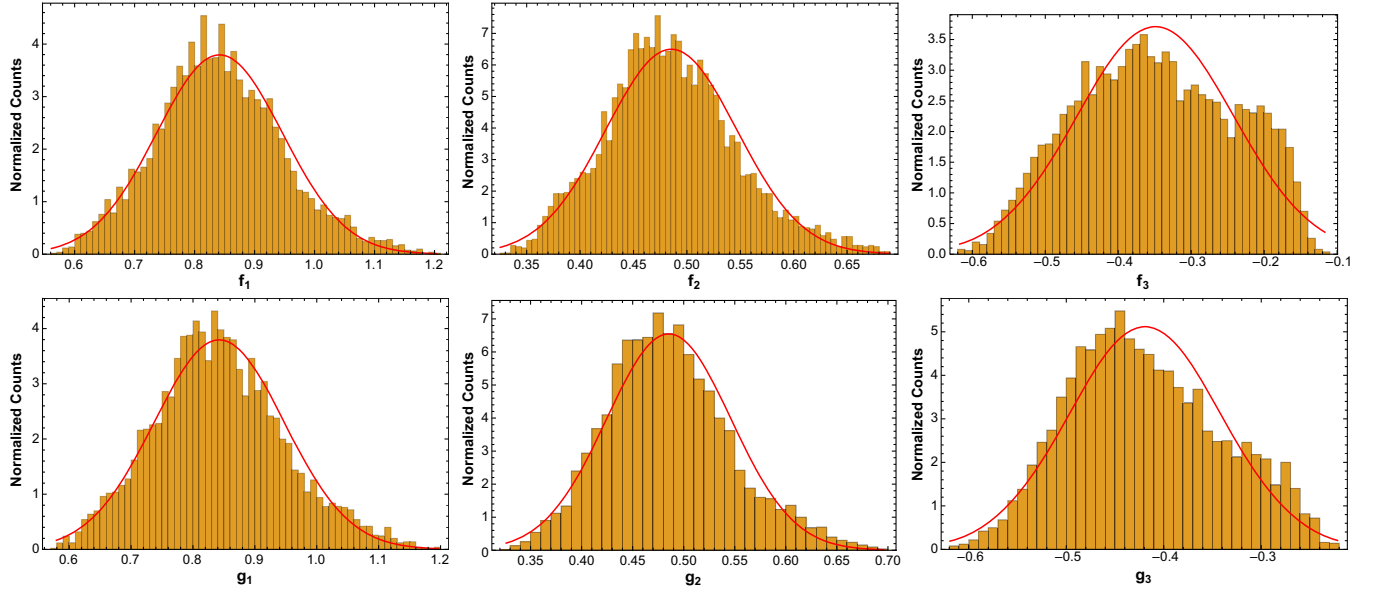


FIG. 1. Normalized distributions of the $\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu$ form factors f_i and g_i at $q^2 = 0$ obtained from LCSR. The solid lines represent Gaussian fits to the Monte Carlo distributions.

The uncertainties of the fit parameters are estimated through a Monte Carlo simulation. We generated 5000 pseudoexperiments by randomly sampling the input parameters within their uncertainties, and computed the corresponding form factors at $q^2 = 0$ in each case. As an illustration, Fig. 1 shows the distribution of the form factor values obtained from the ensemble. The resulting distributions were fit with Gaussians to extract central values and standard deviations, which are quoted as the uncertainties in Table II. After determining the form factors, we compute the total decay widths and branching ratios using Eq. (27). The lifetimes of the charmed baryons required for these calculations are taken from Table I.

Based on the fitted form factors and baryons lifetimes, the branching ratios for the semileptonic decays are computed as follows:

$$\begin{aligned} \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) &= (3.73 \pm 1.04)\%, \\ \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) &= (3.59 \pm 1.01)\%, \\ \mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e) &= (11.20 \pm 3.25)\%, \\ \mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 \mu^+ \nu_\mu) &= (10.8 \pm 3.13)\%. \end{aligned}$$

We note that, in estimating the form factors and the corresponding branching ratios, we also calculated the form factors obtained from alternative Lorentz structures. Our numerical analysis shows that the resulting branching ratios are very close to each other; in particular, the differences are below 4%.

As has already been discussed, semileptonic Ξ_c decays have been examined using various theoretical frameworks. For comparison, Table III compiles the predicted branching

ratios from these approaches along with available experimental measurements. From this comparison, we make the following observations: Our prediction for the branching ratio of $\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$ shows good agreement with the recent lattice QCD result [4], but deviates from other theoretical predictions—including LCSR results based on the distribution amplitudes of Ξ baryons [18,19] as well as from current experimental measurements.

The difference with our previous LCSR calculation [19] can be explained as follows. The principal nonperturbative inputs in Ref. [19] and in the present work are the distribution amplitudes of the Ξ and Ξ_c baryons, respectively. The observed discrepancy between the results of these studies indicates that a more accurate determination of the distribution amplitudes, as well as a more precise evaluation of the parameters entering them, is required.

For completeness, we note that the first lattice QCD calculation [9] also predicts a branching fraction above the experimental value, though below the most recent lattice result. Thus, even within lattice QCD, there is a clear spread between the first and the latest determinations, both of which lie significantly above the Belle II result.

These findings underscore a possible tension between theoretical predictions and experimental data for the $\Xi_c \rightarrow \Xi \ell \nu$ channels. This discrepancy calls for both improved experimental measurements and refined theoretical approaches. There are two possible sources that could help clarify the situation:

- (a) As previously noted, the branching ratio $\Xi_c \rightarrow \Xi \ell \nu$ is often inferred using experimental data for the non-leptonic decay $\Xi_c \rightarrow \Xi \pi$. Recent SU(3) flavor symmetry analyses of charm decays suggest that the measured value of $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)$ may be underestimated [32,33].

TABLE III. Existing experimental and theoretical results for the branching ratios (in %) of the semileptonic $\Xi_c \rightarrow \Xi \ell \nu$ decays.

	$\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$	$\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu$	$\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e$	$\Xi_c^+ \rightarrow \Xi^0 \mu^+ \nu_\mu$
<i>This Work</i>	3.73 ± 1.04	3.59 ± 1.01	11.2 ± 3.25	10.8 ± 3.13
LCSR [19]	1.85 ± 0.56	1.79 ± 0.54	5.51 ± 1.65	5.34 ± 1.61
BELLE II [2]	$1.72 \pm 0.10 \pm 0.12 \pm 0.50$	$1.71 \pm 0.17 \pm 0.13 \pm 0.50$	—	—
ALICE [3]	1.8 ± 0.2	1.8 ± 0.2	—	—
SU(3) [11]	4.87 ± 1.74	—	$3.38_{-2.26}^{+2.10}$	—
SU(3) [8]	2.4 ± 0.3	2.4 ± 0.3	9.8 ± 1.1	9.8 ± 1.1
RQM [29]	2.38	2.31	9.40	9.11
LATTICE [9]	2.38 ± 0.33	$2.29 \pm 0.29 \pm 0.33$	$7.18 \pm 0.90 \pm 0.98$	$6.91 \pm 0.87 \pm 0.93$
LATTICE [4]	3.58 ± 0.12	3.47 ± 0.12	10.94 ± 0.34	10.61 ± 0.33
3PSR [16]	1.45 ± 0.31	1.45 ± 0.31	—	—
LCSR [18]	7.26 ± 2.54	7.15 ± 2.50	28.6 ± 10	28.2 ± 9.9
LFQM [30]	1.354	—	5.39	—
LF [31]	1.72 ± 0.35	—	5.2 ± 1.02	—

Therefore, a new precise measurement of this decay mode is essential for resolving the tension.

- (b) A more precise determination of the distribution amplitudes would significantly enhance the reliability of QCD-based predictions, including those obtained via light-cone sum rules.

IV. CONCLUSION

In this work, we presented a new LCSR analysis of the semileptonic decays $\Xi_c \rightarrow \Xi \ell^+ \nu_\ell$, using the distribution amplitudes of the initial Ξ_c baryon as the primary nonperturbative input. Our predictions for the branching ratios—particularly for the neutral channel $\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu$ —are notably larger than current experimental measurements, yet they are in good agreement with recent lattice QCD results. This consistency suggests a possible tension between theoretical predictions and experimental

extractions, which often rely on indirect determinations via normalization to hadronic decay modes. A remeasurement of $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)$ or a direct measurement of $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu)$ would help resolve this ambiguity, while a more precise determination of the distribution amplitudes of heavy baryons would provide a firmer benchmark for theoretical frameworks. Overall, our results show that further experimental and theoretical investigations are necessary to improve our understanding of semileptonic charmed baryon decays.

DATA AVAILABILITY

This work is a theoretical study. All numerical results are obtained from analytical expressions using input parameters taken from the literature. No new datasets were generated.

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